

Quantum Generation of Dark Energy

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We present a type of dark energy models where the particles of dark energy ϕ are dynamically produced via a quantum transition at very low energies. The scale where the transition takes place depends on the strength g of the interaction between ϕ and a relativistic field φ . We show that a $g \simeq 10^{-12}$ gives a generation scale $E_{gen} \simeq 1 \text{ eV}$ with a cross section $\sigma \simeq 1 \text{ pb}$ close to the WIMPs cross section $\sigma_w \simeq \text{pb}$ at decoupling. The number density n_ϕ of the ϕ particles is a source term in the equation of motion of ϕ that generates the scalar potential $v(\phi)$ responsible for the late time acceleration of our universe. Since the appearance of ϕ may be at very low scales, close to present time, the cosmological coincidence problem can be explained simply due to the size of the coupling constant. In this context it is natural to unify dark energy with inflation in terms of a single scalar field ϕ . We use the same potential $v(\phi)$ for inflation and dark energy. However, after inflation ϕ decays completely and reheats the universe at a scale $E_{RH} \propto h^2 m_{Pl}$, where h is the coupling between the SM particles and φ . The field ϕ disappears from the spectrum during most of the time, from reheating until its re-generation at late times, and therefore it does not interfere with the standard decelerating radiation/matter cosmological model allowing for a successful unification scheme. We show that the same interaction term that gives rise to the inflaton decay accounts for the late time re-generation of the ϕ field giving rise to dark energy. We present a simple model where the strength of the g and h couplings are set by the inflation scale E_I with $g = h^2 \propto E_I/m_{Pl}$ giving a reheating scale $E_{RH} \propto E_I$ and ϕ -generation scale $E_{gen} \propto E_I^2/m_{Pl} \ll E_{RH}$. With this identification we reduce the number of parameters and the appearance of dark energy is then given in terms of the inflation scale E_I .

I. INTRODUCTION.

The nature and dynamics of Dark Energy "DE", which gives the accelerating expansion of the universe at present time, is now days one of the most interesting and stimulating fields of physics. It was discovered more than ten years ago [1] and it has been confirmed by further cosmological observations, being now one of the most robust conjecture in modern physics. In fact the acceleration of the present universe has many experimental proofs such as the CMB temperature and fluctuations [4], in the matter power spectrum measured by galaxy surveys [5, 6] and in type Ia supernovae [7, 8, 9].

The most popular model is the so called Λ CDM model, in which a cosmological constant and some amount of cold dark matter are included "by hand". Despite its extraordinary consistence with observations, Λ CDM is an effective model that leaves many unsolved theoretical question. In fact the existence of the cosmological constant and its order of magnitude have no theoretical justification in Λ CDM. The cosmological coincidence problem, that is why the universe is starting to accelerate right now, is also unsolved. Introducing a cosmological constant at the initial stages of the standard cosmological model is specially troublesome since one has to fine tune its value to one parte in 10^{120} . This problem can be ameliorated if we understand when and how dark energy appears in the universe and this is the main motivation of our present work [17]. However, our approach will also help us to unify dark energy with inflation.

An attractive dark energy alternative to the Λ CDM model consists in introducing a "quintessence" scalar field ϕ that generates the accelerating expansion [13][14] of the universe due to its dynamics. The dynamics is fixed by its potential $v(\phi)$ and it is possible to choose potentials that lead to a late time acceleration of the universe [14]. This scalar field can be a fundamental or composite particle as for example bound states [27]. In the second case, the bound quintessence fields are scalar fields composed of fundamental fermions, such as meson fields, and can be generated at low energies as a consequence of a low phase transition scale due to a strong gauge coupling constant [27]. This allows to understand why DE appears at such late times. On the other hand, in the former case the appearance of fundamental scalar field is right at the beginning of the reheated universe and the acceleration of the universe takes place at a much later time due to the classical evolution of the quintessence field ϕ . The huge difference in scales between the reheating and dark energy scales requires a fine tuning in the choice of the potential.

Here, we present an interesting alternative, namely that the emergence of the fundamental quintessence particles ϕ is originated from a quantum transition taking place at low energies, e.g. as low as eV [17]. The scale where this transition takes place depends on the strength of the interaction between ϕ and a relativistic field φ and it is dynamically determined by the ratio Γ/H where Γ is the transition rate and H the Hubble constant. A value of the coupling $g \simeq 10^{-12}$ gives

a generation scale $E_{gen} \simeq 1 \text{ eV}$ with a cross section $\sigma = g^2/32\pi E_{gen} \simeq 1 \text{ pb}$ close to the WIMPs cross section $\sigma_w \simeq O(\text{pb})$ at decoupling[16]. The subsequent acceleration of the universe is due to the classical evolution of ϕ due to the scalar potential $v(\phi)$. Our quantum generation scheme does not aim to derive the potential $v(\phi)$ but to understand why dark energy dominates at such a late time. Clearly, by closing the gap between the energy today E_o , where the subscript o always refers to present time quantities, and that of ϕ production E_{gen} , we do not require a fine tuning of the parameters in $v(\phi)$. Since the appearance of ϕ may be at such low scales this offers a new interpretation and solution to the cosmological coincidence problem in terms of the size of the coupling constant g .

Furthermore, this late time production of the ϕ particles allows to implement in a natural way a dark energy-inflaton unification scheme. In this scenario, after inflation the field ϕ decays completely and reheats the universe with standard model particles. The universe expands then in a decelerating way dominated first by radiation and later by matter. At low energies the same interaction term that gives rise to the inflaton decay accounts for the quantum re-generation of the ϕ field giving rise to dark energy. In general, it is not complicated to choose a scalar potential such that the universe accelerates in two different regions, at early inflation and dark energy epochs, as in quintessential models [15]. However, the universe requires to be most of the time dominated first by standard model relativistic particles and later by matter. The reheating of the universe and the long period of decelerating phase are usually not taken into account in inflation-dark energy unification models and these features are essential in the standard cosmological Big Bang model. In our case, the inflaton-dark energy field is completely absent during most of the time (from reheating until re-generation) and therefore it does not interfere with the standard cosmological model. We will exemplify our inflation-dark energy unified scheme with a simple model. The scalar potential $v(\phi)$ will have only two parameters fixed by the conditions to give the correct density perturbations $\delta\rho/\rho$ and the present time dark energy scale. The two couplings h, g , which give the strength of reheating process with SM particles and the ϕ re-generation process at low energies, respectively, are free parameters but we may take them as $g = h^2 \propto E_I/m_{pl}$, where E_I is the scale of inflation and it is one of the parameters of $v(\phi)$. Therefore, starting with four free parameters we can reduce the number to only two and these two are fixed by observations. This gives a reheating scale $E_{RH} \propto E_I$ and ϕ -generation scale $E_{gen} \propto E_I^2/m_{pl} \ll E_{RH}$.

The paper is organized as follows: in section II we give an overview of the late time quantum generation of the quintessence field ϕ and its possible unification with the inflaton field. In section III we present the dark energy quantum generation in detail. In section III we show how to unify inflation and dark energy in terms of a sin-

gle scalar field in the context of our dark energy quantum generation process and we present a simple model. Finally, in section V we present the main phenomenological consequences of our model while in section VI we resume and conclude.

II. OVERVIEW

To avoid any future confusion we state the terminology used in this work. We define as usual the energy of the universe at any given time as $E \equiv \rho^{1/4}$ or for a i -species as $E_i \equiv \rho_i^{1/4}$. For particles we take their energy E_a as $E_a = (p_a^2 + m_a^2)^{1/2}$, where p_a, m_a are the momentum and mass of the a -particle. If the particles are relativistic and in thermal equilibrium then one can define a temperature T with an energy density $\rho_a = \frac{\pi^2}{30} g_a T^4$ and number density $n_a = \zeta(3)/\pi^2 g_a T^3$, with g_a the relativistic degrees of freedom and $\zeta(3) \simeq 1.2$ is the Riemann zeta function of 3. The energy distribution of thermalized particles is strongly peaked around the mean value $\bar{E} = \bar{\epsilon} T$ with $\bar{\epsilon} \equiv (\rho/nT) = \pi^4/30\zeta(3) \simeq 2.7$ and we identify the energy of the particle E_a with the average energy $E_a = \bar{E}_a = \bar{\epsilon} T$. We will loosely refer then to either the temperature or the energy of the (thermalized) relativistic particles and we will work in natural units with $m_{pl}^2 \equiv 1/8\pi G = 1$ but we will reintroduce the correct energy dimensions when convenient.

We will first present the quantum generation process of dark energy field ϕ and we will later discuss the possibility to unify it with the inflaton field. As mentioned in the introduction, the main goal of this paper is to describe how the dark energy field ϕ may be generated at a very late time via a quantum transition process and how the scalar potential $v(\phi)$, responsible for the late time acceleration, is generated. Once $v(\phi)$ is produced, the classical equation of motion gives the dynamics of ϕ , and choosing an appropriated flat $v(\phi)$ at low energies ensures that the universe enters an accelerating epoch close to present time. This work does not aim to derive the potential $v(\phi)$ but to understand why dark energy dominates at such a late time. Of course, by closing the gap between the energy today E_o and that of ϕ production E_{gen} there is no longer a fine tuning of the parameters in $v(\phi)$.

We describe now the generation process. We take a universe filled with standard model SM particles and an extra relativistic field φ and no ϕ particles, i.e. $\Omega_\phi = 0$. The φ particle is not necessarily contained in the SM (however, an interesting possibility is to associate φ with neutrinos) but we require that at time of ϕ -generation $\rho_\varphi(E_{gen}) > \rho_{DE}(E_o)$. In the context of inflation-dark energy unification the φ must have been in thermal equilibrium with the SM and therefore $T_\varphi \simeq T_\gamma$ with $\Omega_\varphi \simeq (g_\varphi/g_{rel}^{SM})\Omega_{rel}^{SM}$, where g_φ and g_{rel}^{SM} are the relativistic degrees of freedom of φ and the SM respectively and Ω_{rel}^{SM} is the density of SM relativistic particles. Since we want to produce the ϕ particles via a quantum tran-

sition process we couple it to φ via an interaction term L_{int} , e.g. $L_{int} = g\phi\varphi^3$ or $L_{int} = g\phi^2\varphi^2$. In order to produce the ϕ particles at low energies we require that the interaction rate Γ of the quantum process must be initially smaller than the Hubble parameter H and the ratio Γ/H should increase with the expansion of the universe. This will happen if Γ decreases less rapidly than H . For example if we have a $2 \leftrightarrow 2$ process of relativistic particles the transition rate scale as $\Gamma \propto g^2 T_\varphi$ and $H \propto T_\varphi^2$ giving $\Gamma/H \equiv T_{gen}/T_\varphi \propto g^2/T_\varphi$, where we have taken $T_\varphi \simeq T_\gamma$. The ϕ particles production starts then for temperatures T below $T_{gen} \propto g^2$, where $\Gamma/H > 1$. The energy scale at which the ϕ is generated is fixed by the coupling constant g , so the coincidence problem is explained in terms of the strength of the interactions of the ϕ field. In particular one can generate the ϕ field at low energies, e.g. $E_{gen} \simeq 1\text{ eV}$ with $g \simeq 10^{-12}$ giving a fine structure constant $\alpha = g^2/4\pi \simeq 10^{-25}$ and a cross section $\sigma \simeq g^2/32\pi E_{gen}^2 \simeq 1\text{ pb}$. The value of σ is quite close to cross section of WIMP dark matter with nucleons $\sigma_w \simeq \text{pb}$ [16]. Since we can choose the coupling g in such a way that the ϕ is generated at low energies close to present time, this offers a new interpretation of the cosmological coincidence problem: dark energy domination starts at such small energies because of the size of the coupling constant g . For a ϕ -generation energy of $E_{gen} \simeq 1\text{ eV}$ the ratio in energy densities from the appearance of the fundamental field ϕ to present time is $\rho_{DE}/\rho_{gen} = 10^{-12}$ and should be compared to a case where ϕ is present at the Planck time ρ_{pl} with $\rho_{DE}/\rho_{pl} = 10^{-124}$, giving a difference in ratios of 112 orders of magnitude. Therefore, the amount of fine tuning in the parameters of the dark energy potential $v(\phi)$ is much less severe in our case than in a standard quintessence dark energy model. The production of the ϕ particles gives rise to the scalar potential $v(\phi)$ and the field ϕ will then evolve classically given by its equation of motion. Of course, we still need to choose $v(\phi)$ appropriately to give dark energy, for example $v(\phi) = v_{DEo}/\phi$ with v_{DEo} the present time dark energy potential [14]. We stress the fact that the dark energy behavior of the universe is due to the form of the potential $v(\phi)$ but the energy E_{gen} at which the ϕ is generated is fixed by the coupling constant g . An interesting possibility is to unify inflation and dark energy using the same scalar field ϕ in our dark energy quantum generation picture [17]. We assume that ϕ has a potential $v(\phi)$ which gives an early inflation at E_I , as in standard inflationary models [3]. In this case we use the coupling $L_{int} = g\phi\varphi^3$ to allow the ϕ field to decay into φ particles after inflation, via the process $\phi \rightarrow \varphi + \varphi + \varphi$. This decay process is very efficient [21] and the ϕ decays completely disappearing entirely from the spectrum of the universe. In order to produce SM particles, the φ is coupled with the SM via the standard interactions $L_{int} = h\varphi^2\chi^2$ or $L_{int} = \sqrt{h}\varphi\psi\psi$ with χ, ψ SM scalar or fermions, respectively. Reheating of the early universe takes place via a $2 \leftrightarrow 2$ process $\varphi + \varphi \leftrightarrow \chi + \chi$ or $\varphi + \varphi \leftrightarrow \psi + \psi$ with a transition

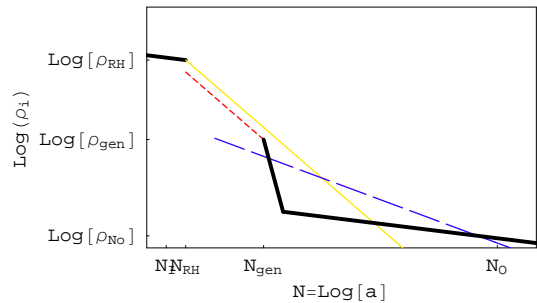


FIG. 1: We plot the logarithms of the energy densities ρ_i , with the density of ϕ field (black line), φ field (red dotted-line), radiation (yellow line) and of matter (blue dashed line) against the number of e-folds $N = \text{Log}[a]$. We see how at N_{RH} the ϕ field disappears and the universe is reheated. At N_{gen} the ϕ field is re-generated and the φ field disappears. We see how the ϕ field is rapidly diluted and then maintains nearly constant at $\rho_\phi \simeq \rho_{DE}$. At late times close to present time, i.e. at $N \simeq N_0$, the ϕ field starts to dominate and inflate the universe.

rate Γ_{RH} , for $\Gamma_{RH}/H \equiv T_{RH}/T_\varphi \propto h^2/T_\varphi > 1$ at an energy scale $E_{RH} \propto h^2$. Because of its couplings, the SM particles and φ are in thermal equilibrium at the time of reheating, and as long as φ remains relativistic one has $\Omega_\varphi \simeq \Omega_{SM} g_\varphi/g_{SM}$. After reheating ρ_φ evolves as radiation and eventually it will re-generate the ϕ field, at an energy $E_{gen} \propto g^2$. The re-generation process of ϕ can be produced with the same interaction $L_{int} = g\phi\varphi^3$ as the inflaton decay at E_I . A main difference in the transition processes at E_I and at E_{gen} is the size of the mass of ϕ . It varies quite significantly and we should have $m_\phi(E_I) \gg m_\phi(E_{gen})$ and also $E_{gen} \gg m_\phi(E_{gen})$. In this case the inflaton-dark energy potential $v(\phi)$ must be chosen to be flat at high energy E_I , to give inflation, and at low energy E_o to accelerate the universe at present time. The reheating energy $E_{RH} \propto h^2$ and the ϕ generation energy $E_{gen} \propto g^2$ are fixed by the coupling constants h and g independently of the potential $v(\phi)$. In the model presented in section IV we set $g = h^2 \propto E_I/m_{pl}$ reducing the number of parameters and connecting the inflation scale to the ϕ -generation and to the reheating scales, $E_{gen} \propto E_I^2/m_{pl}$, $E_{RH} \propto E_I$. As mentioned in the introduction, any unification of inflation and dark energy, such as quintessential models, need to explain how the universe is reheated with SM particles and must account for the long period of decelerating universe. In our model we are able to explain both features in a consistent way.

We give a schematic picture of the inflation-dark energy unified model showing in fig.(1) the evolution of the logarithms of the energy densities of the ϕ and φ fields, radiation and matter. We start with a ϕ dominated universe and then at N_{RH} , where $N \equiv \ln(a)$ with $a(t)$ the scale factor, the ϕ field decays and disappears, reheating the universe. After reheating the universe is radiation

dominated, as in the standard cosmological model, and radiation includes the SM relativistic degrees of freedom plus the extra relativistic field φ . At N_{gen} the ϕ field is re-generated with $\rho_{\phi_f} \simeq \rho_{\varphi_i}$, where ρ_{ϕ_f} is the value of ρ_ϕ at the end of the generation process and ρ_{φ_i} is the value of ρ_φ at the begin of the ϕ generation. At the same time the φ field decays and ρ_{rel} diminishes. After the generation, ρ_ϕ is rapidly diluted and subsequently maintains nearly constant at $\rho_\phi \simeq \rho_{DE}$ and it starts to dominate close to present times, i.e. at $N \simeq N_o$, inflating the universe.

III. THE ϕ GENERATION.

In this section we describe the quantum generation of the quintessence scalar field ϕ and the subsequent appearance of the dark energy behavior of the late universe due to the classical evolution of ϕ . In what follows we assume a universe that consists of the particles of the standard model "SM" together with a massless scalar field φ and a quintessence scalar field ϕ coupled together via a four particle interaction, as for example $L_{int} = g\phi\varphi^3$ or $L_{int} = g\phi^2\varphi^2$. The φ field is also coupled with the SM through the interactions $\sqrt{h}\varphi\psi\psi$ or $h\varphi^2\chi^2$, where ψ and χ are some SM fermions or scalars respectively. We assume that at temperatures above 1TeV all the SM particles and φ are relativistic and φ is in thermal equilibrium with $T_\varphi = T_{rad}$. As long as φ is relativistic $T_\varphi \simeq T_{rad}$ and $\Omega_\varphi \simeq \Omega_{rad}^{SM} \frac{g_\varphi}{g_{SM}^{SM}}$, where Ω_{rad}^{SM} is the SM radiation density and g_φ, g_{SM}^{SM} are the relativistic degree of freedom of φ and SM, respectively. We also suppose that at temperatures $T \gg 1\text{eV}$ the universe does not contain any ϕ particles, therefore the number density of ϕ particles is zero $n_\phi = \Omega_\phi = 0$.

We will show that the field ϕ is generated via the quantum transition $\varphi + \varphi \rightarrow \varphi + \phi$ or $\varphi + \varphi \rightarrow \phi + \phi$ with a transition rate $\Gamma_{\varphi+\varphi \rightarrow \varphi+\phi} = \Gamma_{\varphi+\varphi \rightarrow \phi+\phi} = \langle \sigma_{gen} v \rangle n_\varphi \equiv \Gamma_{gen}$, where $\sigma_{gen} = g^2/32\pi E^2$ is the cross section for a $2 \leftrightarrow 2$ relativistic particle process, v is the relative velocity and n_φ is the density number of the φ particles [25]. This take place at energies below $E_{gen} \simeq c_{gen} g^2 m_{pl}$ with c_{gen} a constant, when $\Gamma_{gen}/H > 1$. At the end of the ϕ generation one has $\Omega_{\phi_f} \simeq \Omega_{\varphi_i}$, where Ω_{φ_i} is the φ density at the begin of the ϕ particles production, and also $\Omega_{\varphi_f} \ll \Omega_{\varphi_i}$. The production of relativistic particles ϕ becomes a source term for the generation of the scalar potential $v(\phi)$. Once $v(\phi)$ has been produced the classical equation of motion gives the evolution of ϕ . The acceleration of the universe is then due to the form of scalar potential $v(\phi)$.

We consider a system composed by two scalar fields ϕ and φ plus SM, in a flat FRW metric, with a density lagrangian

$$L = \frac{\partial_\mu \phi \partial^\mu \phi}{2} + \frac{\partial_\mu \varphi \partial^\mu \varphi}{2} - V_T(\phi, \varphi) + L_{SM} \quad (1)$$

where L is the SM lagrangian and $V_T(\phi, \varphi) = v(\phi) +$

$B(\varphi) + v_{int}(\phi, \varphi)$, $v(\phi)$ and $B(\varphi)$ are the classical potentials of the two scalar fields ϕ and φ and $v_{int}(\phi, \varphi) = -L_{int}(\phi, \varphi)$, where L_{int} is the interaction lagrangian. L_{int} plays a double role: it affects the classical evolution of the two scalar fields and it also originates the quantum transitions between ϕ and φ particles that we use to generate the ϕ field at late times. We divide the $\phi(t, x)$ field into a classical background configuration $\phi_c(t)$ plus a perturbation $\delta\phi(t, x)$ as $\phi(t, x) = \phi_c(t) + \delta\phi(t, x)$, where $\delta\phi(t, x)$ corresponds to the quantum configuration of the ϕ field (ϕ particles). The background $\phi_c(t)$ and the perturbation $\delta\phi(t, x)$ are usually taken as independent variables. However we choose to take as the two independent variables $\phi(t, x)$ and $\delta\phi(t, x)$. Moreover we express $\delta\phi(t, x)$ in terms of the number density n_ϕ of ϕ particles via the relation $n_\phi = E_\phi \delta\phi^2$, see appendix A. We stress the fact that when $n_\phi = 0$ one also has $\delta\phi(t, x) = 0$ that implies $\phi(t, x) = \phi_c(t)$, so the scalar field ϕ is in its classical configuration. Following the same argument we write $\varphi(t, x) = \varphi_c(t) + \delta\varphi(t, x)$ and describe the φ field through the variables $\varphi(t, x)$ and $n_\varphi = E_\varphi \delta\varphi^2$. Let us concentrate on the process of ϕ particles production in the case that both ϕ and φ particles are relativistic and the number density of ϕ particles n_ϕ is initially zero. We use a four particles interaction, such as $L_{int} = g\phi\varphi^3$ or $L_{int} = g\phi^2\varphi^2$ and we consider a $2 \leftrightarrow 2$ the processes, e.g. $\varphi + \varphi \leftrightarrow \varphi + \phi$ or $\varphi + \varphi \leftrightarrow \phi + \phi$. As mentioned previously, the φ field is initially thermalized and it has a phase space distribution given by the Bose-Einstein distribution $f_\varphi(E) = 1/(e^{E/T_\varphi} - 1)$, so we can take all the φ particles with the same energy $E_\varphi = \bar{\epsilon} T_\varphi \simeq T_{rad}$. Since the $2 \leftrightarrow 2$ interaction process conserves energy the ϕ particles become in thermal equilibrium with φ and they has the same energy $E_\phi = E_\varphi = E$. Moreover the two scalar fields ϕ and φ are relativistic so their energy scales as $E = E_i/a(t)$. Before the ϕ generation the density number of φ particles evolves as $n_\varphi = \zeta(3)T_\varphi^3/\pi^2$, so the transition rate for the considered $2 \leftrightarrow 2$ processes of relativistic particles is expressed in terms of the energy of the quantum particles as $\Gamma_{gen} = \langle \sigma_{gen} v \rangle n_\varphi = (\zeta(3)/32\pi^3 \bar{\epsilon}^3) g^2 E$ and therefore it scales as $\Gamma_{gen} \sim 1/a(t)$. As discussed in section II, in order to have a late time dark energy generation, it is fundamental to have a ratio Γ_{gen}/H that grows up with the expansion of the universe. If we are in radiation domination $H \sim a(t)^{-2}$ in in matter domination $H \sim a(t)^{-3/2}$. This means that the Γ_{gen} should decrease more slowly than $1/a(t)^2$ during radiation domination and than $1/a(t)^{3/2}$ during matter domination, as in our case. Taking a flat FRW metric, the evolution of our system is described by the following equations (see eqs.(A20) in appendix A for details)

$$\dot{n}_\phi + 3Hn_\phi = \tilde{\Gamma}(n_\varphi - n_\phi) + 2\sqrt{E n_\phi} \dot{\phi} \quad (2)$$

$$\dot{n}_\varphi + 3Hn_\varphi = -\tilde{\Gamma}(n_\varphi - n_\phi) + 2\sqrt{E n_\varphi} \dot{\varphi} \quad (3)$$

$$\ddot{\phi} + 3H\dot{\phi} + E^{3/2}\sqrt{n_\phi} + \partial_\phi v(\phi) + \partial_\phi v_{int}(\phi, \varphi) = 0 \quad (4)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + E^{3/2}\sqrt{n_\varphi} + \partial_\varphi B(\varphi) + \partial_\varphi v_{int}(\phi, \varphi) = 0 \quad (5)$$

together with the Friedman equation $H^2 = \frac{1}{3}\rho_T$, where

$\rho_T = \rho_\phi + \rho_\varphi + \rho_{rad} + \rho_{mat}$, and ρ_{rad} and ρ_{mat} are the energy densities of radiation and matter respectively. Moreover $v(\phi)$ and $B(\varphi)$ are the classical potentials of the ϕ and φ fields respectively and $v_{int}(\phi, \varphi)$ is the classical interaction potential.

Eqs.(2) and (3) are the Boltzmann equations that govern the dynamic of the number densities n_ϕ and n_φ . They take into account the quantum transition between ϕ and φ particles thanks to the terms proportional to $\tilde{\Gamma}$. The quantity $\tilde{\Gamma}$ in eqs. (2) and (3) is given by $\tilde{\Gamma} \equiv \langle \sigma_{gen} v \rangle n_\varphi$ for the process $\varphi\varphi \leftrightarrow \varphi\phi$ and $\tilde{\Gamma} \equiv \langle \sigma_{gen} v \rangle (n_\varphi + n_\phi)$ for the process $\varphi\varphi \leftrightarrow \phi\phi$. $\tilde{\Gamma}$ takes into account the contribution of the two processes $\varphi\varphi \rightarrow \varphi\phi$ and its inverse $\varphi\phi \rightarrow \varphi\varphi$ in one case and $\varphi\varphi \rightarrow \phi\phi$ and $\phi\phi \rightarrow \varphi\varphi$ in the other case, since one has $\Gamma_{\varphi+\varphi \rightarrow \varphi+\phi} n_\varphi - \Gamma_{\varphi+\phi \rightarrow \varphi+\varphi} n_\phi = \langle \sigma_{gen} v \rangle n_\varphi (n_\varphi - n_\phi) = \tilde{\Gamma} (n_\varphi - n_\phi)$ for $\varphi + \varphi \leftrightarrow \varphi + \phi$ and $\Gamma_{\varphi+\varphi \rightarrow \phi+\phi} n_\varphi - \Gamma_{\phi+\phi \rightarrow \varphi+\varphi} n_\phi = \langle \sigma_{gen} v \rangle (n_\varphi^2 - n_\phi^2) = \tilde{\Gamma} (n_\varphi - n_\phi)$ for $\varphi + \varphi \leftrightarrow \phi + \phi$. At the begin of the ϕ generation one has $n_\phi = 0$ and therefore $\tilde{\Gamma} = \Gamma_{gen}$ in both processes. Moreover the two terms $2\sqrt{E_i n_\phi} \dot{\phi}$ and $2\sqrt{E_i n_\varphi} \dot{\varphi}$ contained in eqs.(2) and (3) respectively, couple the number densities n_ϕ and n_φ with the scalars ϕ and φ respectively. Eqs.(4) and (5) are the equations of motion of the lagrangian in eq.(1) for the scalar fields $\phi(t, x)$ and $\varphi(t, x)$. These equations contains the terms $E^{3/2} \sqrt{n_\phi}$ and $E^{3/2} \sqrt{n_\varphi}$ that couple the two scalar fields ϕ and φ with their number densities n_ϕ and n_φ and become the source terms for generating the scalar potential $v(\phi)$. Note that the terms $E^{3/2} \sqrt{n_\phi}$ and $E^{3/2} \sqrt{n_\varphi}$ contained in eqs.(4) and (5) represent the spatial derivatives of the two scalar fields, in fact they come out from the relations $-\frac{\nabla^2 \phi}{a(t)} = E^{3/2} \sqrt{n_\phi}$ and $-\frac{\nabla^2 \varphi}{a(t)} = E^{3/2} \sqrt{n_\varphi}$, see eq.(A4) in appendix A. Of course eqs.(2)-(5) conserve energy-momentum. The energy density and pressure of the ϕ field are $\rho_\phi = \rho_{1\phi} + \rho_{2\phi}$ and $p_\phi = p_{1\phi} + p_{2\phi}$, where $\rho_{1\phi} \equiv \frac{\dot{\phi}^2}{2} + v(\phi)$, $\rho_{2\phi} \equiv E \frac{n_\phi}{2}$, $p_{1\phi} \equiv \frac{\dot{\phi}^2}{2} - v(\phi)$ and $p_{2\phi} = \rho_{2\phi}/3$ (see appendix A). In the same way the energy density and the pressure of the φ field are $\rho_\varphi = \rho_{1\varphi} + \rho_{2\varphi}$ and $p_\varphi = p_{1\varphi} + p_{2\varphi}$, where $\rho_{1\varphi} \equiv \frac{\dot{\varphi}^2}{2} + v_{int}(\phi, \varphi) + B(\varphi)$, $\rho_{2\varphi} \equiv E \frac{n_\varphi}{2}$, $p_{1\varphi} \equiv \frac{\dot{\varphi}^2}{2} - v_{int}(\phi, \varphi) - B(\varphi)$ and $p_{2\varphi} = \rho_{2\varphi}/3$. It is easy to show (see appendix A) that

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = \frac{1}{2} E \tilde{\Gamma} (n_\varphi - n_\phi) - \dot{\phi} \partial_\phi v_{int} \quad (6)$$

$$\dot{\rho}_\varphi + 3H(\rho_\varphi + p_\varphi) = -\frac{1}{2} E \tilde{\Gamma} (n_\varphi - n_\phi) + \dot{\phi} \partial_\phi v_{int} \quad (7)$$

Therefore eqs.(6) give the energy-momentum conservation law $\dot{\rho}_T + 3H(\rho_T + p_T) = 0$, where $\rho_T = \rho_\phi + \rho_\varphi$ and $p_T = p_\phi + p_\varphi$. Now we are ready to describe the ϕ particles production qualitatively. For presentation purposes we will not take into account the expansion of the universe, i.e. we will take $H = 0$, however we show below that the growth of the scale factor is very small (of the order of H/Γ_{gen}) [40]. Moreover in this presentation we will not consider the contribution from classical interactions and we also take $B(\varphi) = 0$. We will also

work with $\tilde{\Gamma}$ constant. Obviously $\tilde{\Gamma}$ depends both on the energy of the decaying particles and on the number densities, but we use this approximation to write an analytical solution of eqs.(2)-(5). Of course comparison of the approximated analytical solution with the numerical solution shows a complete agreement and the classical interaction $v_{int}(\phi, \varphi)$ and expansion rate do not play a significant role. Under these approximations the system of eqs.(2-5) reduce to the following equations

$$\dot{n}_\phi = \tilde{\Gamma} (n_\varphi - n_\phi) + 2\sqrt{E_i n_\phi} \dot{\phi} \quad (8)$$

$$\dot{n}_\varphi = -\tilde{\Gamma} (n_\varphi - n_\phi) + 2\sqrt{E_i n_\varphi} \dot{\varphi} \quad (9)$$

$$\ddot{\phi} + E_i^{3/2} \sqrt{n_\phi} + \partial_\phi v(\phi) = 0 \quad (10)$$

$$\ddot{\varphi} + E_i^{3/2} \sqrt{n_\varphi} = 0 \quad (11)$$

where E_i is the energy of the quantum particles at the begin of the ϕ generation. The initial conditions on the φ field are $\rho_{\varphi_i} = E_i n_{\varphi_i}$, $n_{\varphi_i} = \frac{s(3)}{\pi^2 r^3} E_i^3$ with $\langle \dot{\varphi}_i^2 \rangle = E_i n_{\varphi_i}$ and $\Omega_{\varphi_i} = 0.1$. For the ϕ field we have the initial conditions $\rho_{\phi_i} = 0$ that gives $\Omega_{\phi_i} = n_{\phi_i} = \dot{\phi}_i = v(\phi_i) = 0$.

Now, the first stage starts with $(n_\varphi - n_\phi) \tilde{\Gamma} \gg -2\sqrt{E_i n_\phi} \dot{\phi}$ in eq.(8), which is clearly verified at the begin since $n_{\phi_i} = 0$, and the approximate solution of eqs.(8-11) is

$$\begin{aligned} n_\phi &\simeq n_{\varphi_i} \tilde{\Gamma} (t - t_i) \left[1 - \frac{4}{9} E_i^2 (t - t_i)^2 \right], \\ n_\varphi &\simeq n_{\varphi_i} \left[1 + (2E_i - \tilde{\Gamma}) (t - t_i) \right], \\ \dot{\phi} &\simeq -\frac{2}{3} E_i^{3/2} \sqrt{n_{\varphi_i} \tilde{\Gamma}} (t - t_i)^{3/2}, \\ \dot{\varphi} &\simeq \dot{\varphi}_i [1 - E_i (t - t_i)]. \end{aligned} \quad (12)$$

valid for $\tilde{\Gamma} (t - t_i) \ll 1$ and $E_i (t - t_i) \ll 1$. This first phase of the generation consists in the growth of n_ϕ due to ϕ particle production. However the growth of n_ϕ ends at some time t_1 , when $(n_\varphi - n_\phi) \tilde{\Gamma} = -2\sqrt{E_i n_\phi} \dot{\phi} > 0$ and $\dot{n}_\phi = 0$. This point is the end of stage one and n_ϕ reaches a maximum value $n_\phi = \bar{n}_\phi \equiv n_\varphi + \beta(1 - (1 + 2n_\varphi/\beta)^{1/2})$ with $\beta = 2E_i \dot{\phi}^2 / \tilde{\Gamma}^2$. At the end of this first stage one also has $n_\varphi/\beta \ll 1$, $n_\varphi(t_1) \simeq n_{\varphi_i}$, $\dot{\varphi}(t_1) \simeq \dot{\varphi}_i$ and $n_\phi(t_1) = \bar{n}_\phi \simeq n_\varphi^2 / 2\beta \ll n_\varphi$.

The second phase of the generation maintains $(n_\varphi - n_\phi) \tilde{\Gamma} + 2\sqrt{E_i n_\phi} \dot{\phi} = 0$ dynamically, therefore from eq.(8) it follows that n_ϕ remains constant at the equilibrium value $\bar{n}_\phi \ll n_{\varphi_i}$. One can check the stability of this solution $n_\phi = \bar{n}_\phi$ analyzing the dynamical equation for small perturbations of n_ϕ around \bar{n}_ϕ that is $\delta \dot{n}_\phi = -\beta \delta n_\phi$ with $\beta = \tilde{\Gamma} - \sqrt{\frac{E_i}{\bar{n}_\phi}} \frac{\dot{\phi}}{2} > 0$, that gives an exponential suppression of perturbations, so the solution $n_\phi = \bar{n}_\phi$ is stable. If n_ϕ is maintained constant at its equilibrium value \bar{n}_ϕ , the relation $2\sqrt{E_i n_\phi} \dot{\phi} = -\tilde{\Gamma} (n_\varphi - \bar{n}_\phi) = \dot{n}_\varphi$ is satisfied dynamically (see eqs.(8) and (9)), so we can write eq.(11) in the form $\ddot{\phi} + \partial_\phi v(\phi) \simeq \frac{1}{2} E_i \tilde{\Gamma} \frac{n_\varphi - \bar{n}_\phi}{\phi}$,

or $\dot{\rho}_{1\phi} = \frac{1}{2} E_i \tilde{\Gamma} (n_\phi - \bar{n}_\phi)$ where $\dot{\rho}_{1\phi} \equiv \frac{\dot{\phi}^2}{2} + v(\phi)$. We take $\dot{\phi}^2 \simeq E n_\phi$, which agrees well with the numerical simulation, and $E \dot{\phi} \sqrt{E n_\phi} \simeq E \dot{\phi}^2 \simeq 2 E \rho_{1\phi}$ where $\rho_{1\phi} \equiv \dot{\phi}^2/2$. Therefore eqs.(8)-(11) are reduced to the following effective equations

$$\begin{aligned} n_\phi &= \bar{n}_\phi, & \dot{n}_\phi &= -\tilde{\Gamma} (n_\phi - \bar{n}_\phi) + 4 \rho_{1\phi}, \\ \dot{\rho}_{1\phi} &= \frac{1}{2} \tilde{\Gamma} E_i (n_\phi - \bar{n}_\phi), & (13) \\ \dot{\rho}_{1\phi} &= -2 E \rho_{1\phi} \end{aligned}$$

whose solutions are

$$\begin{aligned} n_\phi &= \bar{n}_\phi, \\ n_\phi &= n_{\phi_i} \left[e^{-\tilde{\Gamma}(t-t_1)} + 2E_i \frac{e^{-\tilde{\Gamma}(t-t_1)} - e^{-2E_i(t-t_1)}}{2E_i - \tilde{\Gamma}} \right] + \\ &\quad + \bar{n}_\phi \left[1 - e^{-\tilde{\Gamma}(t-t_1)} \right], \\ \rho_{1\phi} &= \frac{E_i}{2} (n_{\phi_i} - \bar{n}_\phi) \left[1 - e^{-\tilde{\Gamma}(t-t_1)} \right] + \\ &\quad + E_i n_{\phi_i} \frac{2E_i (1 - e^{-\tilde{\Gamma}(t-t_1)}) - \tilde{\Gamma} (1 - e^{-2E_i(t-t_1)})}{2(2E_i - \tilde{\Gamma})}, \\ \rho_{1\phi} &= \frac{E_i n_{\phi_i}}{2} e^{-2E_i(t-t_1)}. \end{aligned} \quad (14)$$

From these equations we see that n_ϕ decrease until it reaches the equilibrium at $n_{\phi_f} \simeq n_{\phi_f} \simeq \bar{n}_\phi \ll n_{\phi_i}$ and that $\rho_{1\phi}$ decrease exponentially to zero so at the end of the regeneration one has $\rho_{\phi_f} = E_i \bar{n}_\phi \ll \rho_{\phi_i}$. Moreover $\rho_{1\phi}$ grows until $\rho_{1\phi_f} = \rho_{\phi_i} - E_i \bar{n}_\phi$ so one has $\rho_{\phi_f} = \rho_{\phi_i}$ and in conclusion one has $\Omega_{\phi_f} \simeq \Omega_{\phi_i}$, $\Omega_{\phi_f} \ll \Omega_{\phi_i}$ and $v(\phi) \simeq \rho_{\phi_i}$, that means that all the energy initially stored into the quantum field ϕ has been transferred to the ϕ field in accordance with the total energy conservation.

We solve numerically the system of eqs.(2-5) in the case $v(\phi) = v_i/\phi$. In fig.(2) we show the evolution of n_ϕ/E^3 and in fig.(3) that of n_ϕ/E^3 . The evolution of n_ϕ includes the solutions given in eqs.(12) and (14) showing that n_ϕ starts from zero, it reaches its maximum and then it decreases to its equilibrium value \bar{n}_ϕ . The evolution of n_ϕ is plotted in fig.(3) and we see how n_ϕ decrease exponentially to the asymptotic value \bar{n}_ϕ as in eqs.(12) and (14). In fig. (6) we plot the evolution of the density parameter Ω_ϕ showing that Ω_ϕ goes to zero at the end of the ϕ generation. We stress some important features of the ϕ generation process. The first one is that, in spite of starting with a $v(\phi_i) = 0$, at the end of the process one has generated a potential $v(\phi) \simeq \rho_{\phi_i} \neq 0$. In fig.(4) we show how $v(\phi)$ grows from zero to its maximum value $v(\phi) \simeq \rho_{\phi_i}$. At this point the quantum generation process is completed and the subsequent evolution of ϕ is given by its classical equations of motion. At the same time in fig.(5) we see the evolution of $\dot{\phi}$, where it starts at $\dot{\phi}(t_{geni}) = 0$ and takes negative values (implying the growth of $v(\phi)$) and eventually reaches positive values. The time when $\dot{\phi}(t_{genf}) = 0$ corresponds to the end of ϕ generation and to the maximum of $v(\phi)$. Another feature of the ϕ generation process consists of the existence of an equilibrium value $\bar{n}_\phi \ll n_{\phi_i}$ for the density number of ϕ particles n_ϕ . Therefore after a first phase of growth, n_ϕ saturates at \bar{n}_ϕ and then it is maintained constant. This

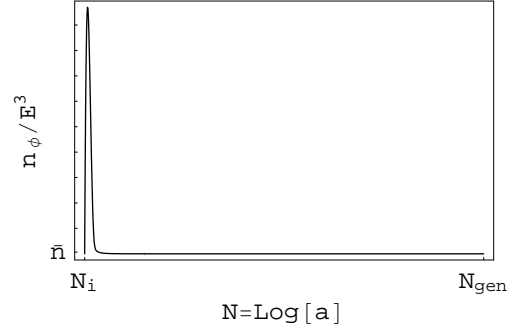


FIG. 2: We show the evolution of n_ϕ/E^3 against the number of e-folds N during the ϕ generation in the case $v(\phi) = v_i/\phi$. We see how n_ϕ grows from zero and then reaches the equilibrium value \bar{n}_ϕ .

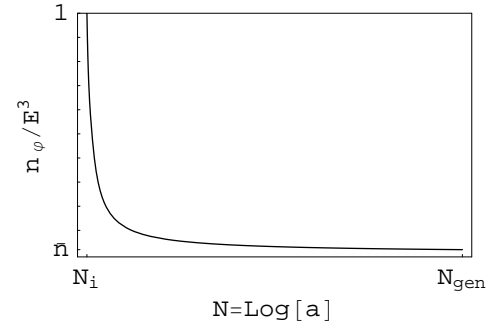


FIG. 3: We show the evolution of n_ϕ/E^3 against the number of e-folds N during the ϕ generation in the case $v(\phi) = v_i/\phi$. n_ϕ decreases exponentially and reaches the asymptotic value \bar{n}_ϕ .

means that all the energy coming from further ϕ particles decay is transferred directly into the potential $v(\phi)$ without generate any change in n_ϕ .

Let us point out more properties of the ϕ generation process. First we note that if the ϕ field is regenerated when the universe is radiation dominated, the scale factor

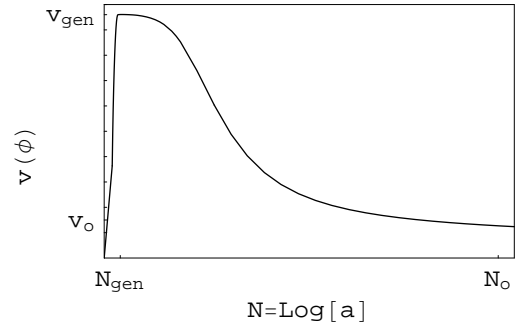


FIG. 4: We show the evolution of the potential $v(\phi)$ against the number of e-folds N in the case $v(\phi) = v_i/\phi$.

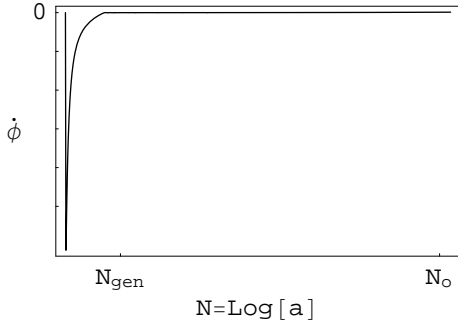


FIG. 5: We show the evolution of the derivative $\dot{\phi}(N)$ against the number of e-folds N in the case $v(\phi) = v_i/\phi$.

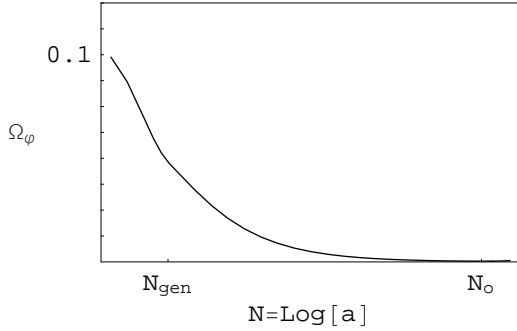


FIG. 6: We show the evolution of Ω_ϕ against the number of e-folds N . We see that Ω_ϕ rapidly goes to zero at the end of the ϕ generation.

evolves as $a(t) = a_i \sqrt{1 + H_i(t - t_i)}$. The ϕ generation ends at $\tilde{\Gamma}(t - t_i) \gtrsim 1$ (see eq.(14)) with $\tilde{\Gamma} \simeq \Gamma_{gen}$, giving $H(t - t_i) \lesssim H/\Gamma_{gen} \leq 1$ and $a(t) \lesssim \sqrt{2}a_i$ (the generation process has $\Gamma_{gen}/H \lesssim 1$), thus we see that the expansion of the universe during the ϕ generation plays no significant role. We also stress the fact that the generation process preserves the homogeneity and isotropy of the universe, as in the reheating process after inflation [3]. In fact it is obtained via a $2 \leftrightarrow 2$ quantum process in which the energy-momentum is conserved ($\sum p_i = \sum p_f$) so the amount of homogeneity is preserved.

Now we can state something more about the begin and the duration of the generation process. First note that at the begin of the ϕ generation $n_\phi = 0$, that implies $\tilde{\Gamma} = \Gamma_{gen} \equiv \Gamma_{\varphi+\varphi \rightarrow \phi+\varphi} = \Gamma_{\varphi+\varphi \rightarrow \phi+\phi}$. Therefore, in order to establish the efficiency of ϕ particles production one should compare the transition rate Γ_{gen} with the expansion rate of the universe H . We assume that before the generation the φ field has the same temperature of the radiation, so we take $E_{rad} = E_\varphi = E$. If the generation starts before the matter domination one has $H = (\rho_{rel}/3m_{pl}^2\Omega_{rel})^{1/2} \equiv c_H E^2$ where $c_H \equiv (\pi^2 g_{rel}/90\bar{r}^4 m_{pl}^2\Omega_{rel})^{1/2}$ and g_{rel} is the total number of relativistic degrees of freedom. There-

fore, using $n_\varphi = \frac{\zeta(3)}{\pi^2} T_\varphi^3$ before the ϕ generation one has $\Gamma_{gen} = \frac{\zeta(3)}{32\pi^3 \bar{r}^3} g^2 E$ and $\Gamma_{gen}/H = c_{gen} g^2 m_{pl}/E$, where we have defined $c_{gen} \equiv (\zeta(3)/32\pi^4 \bar{r}) (90\Omega_{rel}/g_{rel})^{1/2}$. We conclude that the ϕ generation starts at energies $E \lesssim E_{gen}$ with

$$E_{gen} \equiv c_{gen} g^2 m_{pl} = 1.6 \left(\frac{g}{10^{-12}} \right)^2 eV \quad (15)$$

when $\Gamma_{gen}/H \equiv E_{gen}/E \gtrsim 1$. We aim to have a ϕ particles production at energy scales well below MeV , where the number of relativistic degrees of freedom is $g_{rel} = g_{rel}^{SM} + g_\varphi \simeq 4.36$ giving $c_{gen} \simeq 6 \cdot 10^{-4}$. Of course, by means of eq.(15) we can conveniently choose the coupling g in such a way that the generation process starts at an energy of about $1eV$. This takes place if $g \simeq 10^{-12}$ or $\alpha = g^2/4\pi \simeq 10^{-25}$, where α is the fine structure constant of the transition process. We stress the fact that $\Gamma_{gen}/H \equiv E_{gen}/E \geq 1$ during the ϕ generation process but, when the ϕ field starts to inflate the universe, one has $H \sim a(t)^{-\frac{3}{2}(1+\omega_\phi)}$ with $\omega_\phi \simeq -1$, so H will be roughly constant and at the dark energy epoch $\Gamma_{gen}/H \sim a(t)^{\frac{3}{2}(1+\omega_\phi)-1} \rightarrow 0$. Therefore, ϕ and φ fields decouple at $E_{dec} = \frac{\sqrt{3}}{8\pi} \frac{\sqrt{\rho_{DE}}}{g^2 m_{pl}} = \frac{\sqrt{3} c_{gen} E_{DE}^2}{8\pi E_{gen}}$.

We can summarize the ϕ generation process as follows. At the begin of the generation we have $n_{\varphi i} = \frac{\zeta(3)}{\pi^2 \bar{r}^3} E_{\varphi i}^3$, $n_\phi = \Omega_\phi = v(\phi) = 0$ and at $E_{\varphi i} = E_{gen}$ we have $\Gamma_{gen} = H$. The first effect is the decay of φ and the growth of n_ϕ to its equilibrium value $\bar{n}_\phi \ll n_{\varphi i}$ after which n_ϕ remains constant. Subsequently all the energy coming from the φ field is transferred to the potential $v(\phi)$ through the chain reaction $n_\varphi \rightarrow n_\phi \rightarrow v(\phi)$. In this process n_ϕ remains constant and \bar{n}_ϕ represent a maximum transfer efficiency value for n_ϕ in such a way that the energy coming from the decaying φ particles is immediately stored into the potential $v(\phi)$. At the end of the generation one has $n_{\phi f} \simeq n_{\varphi f} \simeq \bar{n}_\phi \ll n_{\varphi i}$, $\rho_{\phi f} \ll \rho_{\varphi i}$ and $\rho_{\phi f} \simeq \rho_{\varphi i}$.

From this point on, the evolution of ϕ is the standard one, where its dynamics is dominated by the classical potential $v(\phi)$. The generation process described before does not depend explicitly on the form of the potential $v(\phi)$ and the value of ϕ at the end of the generation process is given by the condition $v(\phi_{gen}) \simeq \rho_{\varphi i} \simeq E_{\varphi i}^4 = E_{gen}^4$. Since we want the ϕ field to give dark energy, we must choose a potential $v(\phi)$ that slow rolls at late times, i.e. at present time when $\phi \simeq \phi_o$. Clearly we must impose the condition $v(\phi_{gen}) > v(\phi_o) = v_{DE} \simeq (10^{-3} eV)^4$. For example one can consider an effective potential $v(\phi) = \frac{v_i}{\phi}$ that verifies the slow roll conditions for $\phi \geq \sqrt{2}$ and take $v_i = (10^{-3} eV)^4$. In this case the value of ϕ at the end of the generation is $\phi_{gen} \simeq v_i/E_{gen}^4$ where we have used $E_{gen}^4 = v(\phi_{gen}) = \frac{v_i}{\phi_{gen}}$. Typically, runaway quintessence potentials have an EoS parameter ω_ϕ that reaches positive values of $\omega_\phi \simeq 1$ diluting Ω_ϕ and later there is a transition from $\omega_\phi = 1$ to $\omega_\phi = -1$ as in fig.(7) where Ω_ϕ starts growing [14]. Therefore the matter domination epoch is unchanged by our ϕ generation

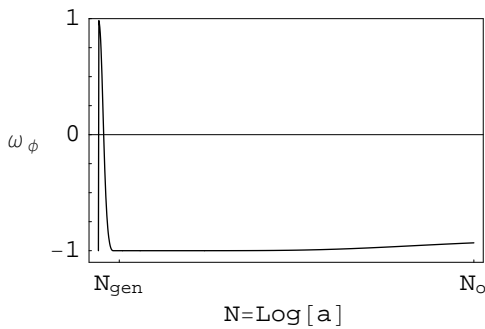


FIG. 7: We show the evolution of the EoS parameter ω_ϕ against the number of e-folds N .

scheme. In fig.(8) we show the evolution of the density parameters Ω_ϕ , Ω_{rad}^{SM} and Ω_{mat} . The ϕ is generated at N_{gen} at energies $E_{gen} \simeq 1\text{eV}$, for $g \simeq 10^{-12}$, close to radiation matter equality with $\Omega_{\phi_i} \simeq 0.2$ and present time is at $\Omega_\phi \simeq 0.74$ and $\Omega_m \simeq 0.26$ with $\omega_\phi \simeq -1$.

The cross section for the generation process is given by

$$\sigma_{gen} = \frac{g^2}{32\pi E_{gen}^2} = 1.5 \left(\frac{10^{-12}}{g} \right)^2 pb = 2.4 \left(\frac{1\text{eV}}{E_{gen}} \right) pb \quad (16)$$

where we have used $E_{gen} \equiv c_{gen} g^2 m_{pl}$ and $c_{gen} \equiv \frac{\zeta(3)}{32\pi^4 r} \sqrt{\frac{90\Omega_{rel}}{g_{rel}}} \simeq 6 \cdot 10^{-4}$ and $pb = 10^{-36} cm^2$. It is interesting to compare the cross section σ_g with that of WIMPS. The relic abundance of WIMPS is $\Omega_w h^2 = 3 \times 10^{-27} cm^3 / < \sigma_w v > [16]$ giving $\langle \sigma_w v \rangle = 0.9 cb$, with c the speed of light. If we take that at decoupling the WIMPS have a mass to temperature ratio $m/T = 20$ [16] we obtain $\langle \sigma_w \rangle = 2.4 pb$ equivalent to our σ_{gen} for $E_{gen} = 1\text{eV}$. However, the present time observational upper limit to σ_w between WIMPS and nucleons is $\sigma_w \lesssim 10^{-42} cm^2$ consistent with supersymmetric WIMPS [16].

IV. UNIFICATION OF INFLATION AND DARK ENERGY.

In this section we discuss the possibility of unifying inflation and dark energy by means of a unique scalar field ϕ that we call uniton, as in [17]. In general it is not difficult to choose the potential $v(\phi)$ in such a way that the ϕ field is responsible for both inflation and dark energy [15]. To achieve inflation and dark energy with the same scalar ϕ one requires that the potential $v(\phi)$ must satisfy the slow roll conditions $|v'(\phi)/v(\phi)| < 1$ and $|v''(\phi)/v(\phi)| < 1$ at high energies for inflation and at low energies for dark energy. Inflationary potentials which have a minimum $v_{min} = 0$ at a finite value of ϕ are not useful to unify inflation and dark energy, since this kind of potentials do not inflate at low energies [14]. This kind of potentials may be useful to unify inflation and

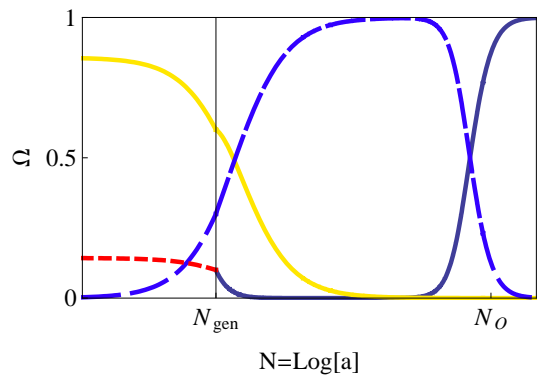


FIG. 8: We plot the density parameters Ω_ϕ (black line), Ω_{rad}^{SM} (yellow line), Ω_{mat} (blue dashed line) and Ω_ϕ (red dotted line) against the number of e-folds $N = \text{Log}[a]$. We see how at N_{gen} the φ disappears and the ϕ field is generated. Present times are at N_o when $\Omega_\phi \simeq 0.74$ and $\Omega_m \simeq 0.26$.

dark matter. In most inflation-dark energy unified models only the classical evolution of the quintessential scalar field is considered and the reheating and the long period of decelerating universe (between inflation and dark energy) are not taken into account.

In this section we present an inflation-dark energy unified scheme that can be resumed in the following way: as in usual inflationary models, the early universe is dominated by the ϕ field that inflates for a sufficient number of e-folds. After the end of inflation the ϕ field decays completely into the extra relativistic field φ already introduced in section III. The φ couples and produces SM particles at an energy E_{RH} and the universe is reheated. At low energies E_{gen} the ϕ field is generated via the quantum generation mechanism studied in section III and the universe enters the dark energy epoch at temperatures close to present time. From E_{RH} to E_{gen} we have the standard evolutionary scenario with the extra relativistic degree of freedom φ .

The new feature in the unification scheme that we present here, is that the transition between decelerating radiation-matter dominate universe and the dark energy era is due to a quantum process, i.e. the low energy generation of the ϕ field. As stated in section III, the energy scale E_{gen} at which the uniton ϕ is generated is fixed by the couplings g of ϕ . The scales E_{gen} may be many orders of magnitude smaller than E_{RH} without any fine tuning.

We stress the fact that, although we will describe the inflation-dark energy unified scheme choosing a particular form of the potential $v(\phi)$, the quantum generation mechanism works well for a large class of potentials. The only requirements on the potential $v(\phi)$ are that it should satisfy the slow roll conditions at high and low energies and that the ϕ mass $m_\phi \equiv \sqrt{v''(\phi)}$ should satisfy the condition $m_\phi(t_{RH}) \gg m_\phi(t_{gen})$, where t_{RH} and t_{gen} are the reheating and ϕ generation times. In addition one

has to require that the ϕ particles are relativistic at t_{gen} , that implies $m_\phi(t_{gen}) \ll E_{gen}$. In section IV A we will discuss the inflation and reheating scenario and then in section IV B we will consider the inflation and dark energy unified scheme using a simple example.

A. Inflation and reheating

Let us consider an universe that contains the field ϕ , a second scalar field φ and the SM particles. We want the ϕ field to inflate the early universe, so we assume that the potential $v(\phi)$ has at least one flat region corresponding to inflation. As an example one can consider the potential described in Appendix B. During inflation the ϕ field dominates the universe and slow rolls as long as the slow roll conditions $|v'(\phi)/v(\phi)| \ll 1$ and $|v''(\phi)/v(\phi)| \ll 1$ are satisfied and the universe inflates with $H^2 \sim v(\phi)$. After the end of inflation the ϕ field decays into φ particles and reheats the universe. We couple the two scalars ϕ, φ via the interaction

$$L_{int} = g \phi f(\varphi) \quad (17)$$

where $f(\varphi)$ is a polynomial of φ . We know that the interaction in eq.(17) gives a complete reheating since it involves a single ϕ particle decaying into φ particles [21]. At the end of inflation the ϕ particles are at rest in the comoving frame (the velocity is redshifted as $v_i = e^{-\Delta N} v_f$) so $E_\phi = m_\phi$ and we take $m_\phi \gg m_\varphi$. We take $f(\varphi) = \varphi^3$ and we consider the process $\phi \rightarrow \varphi + \varphi + \varphi$. For simplicity we assume that all the φ particles are produced with the same energy E_φ given by $E_\phi = 3E_\varphi$ giving a decay rate [2]

$$\Gamma_d = \frac{g^2 m_\phi}{(2\pi)^3 72} \quad (18)$$

Let us remind that a decay process is efficient if $\Gamma_d/H \gtrsim 1$ [3]. The evolution of the energy density ρ_ϕ of the ϕ field is given by the equation

$$\dot{\rho}_\phi + 3H(1 + \omega)\rho_\phi = -\Gamma_d \rho_\phi \quad (19)$$

If we consider ω and Γ_d as piecewise constant, the solution is $\rho_\phi \sim a(t)^{-3(1+\omega)} e^{-\Gamma_d t}$ and the ϕ energy density vanish exponentially, that means that φ particles are produced and the energy of the ϕ field is transferred to the φ field. If one relaxes the hypothesis of constant Γ_d one has $\rho_\phi \sim e^{-\int \Gamma_d dt}$ and the condition for an efficient decay is $\int \Gamma_d dt \gg 1$. At the same time we couple φ with SM particles. We take the usual interaction terms

$$L_{int} = h \varphi^2 \chi^2, \quad L_{int} = \sqrt{h} \varphi \bar{\psi} \psi \quad (20)$$

where χ and ψ are SM scalars and fermions, respectively. As long as SM particles are relativistic, valid at temperatures above $1 TeV$, the processes $\varphi + \varphi \leftrightarrow \chi + \chi$ or $\varphi + \varphi \leftrightarrow \bar{\psi} + \psi$ given by the interaction in eq.(20)

have a transition rate $\Gamma_{RH} = h^2 n_\varphi / 32\pi E_\varphi^2$, and using $n_\varphi = \zeta(3) T_\varphi^3 / \pi^2$ one has $\Gamma_{RH} = \zeta(3) h^2 E_\varphi / 32\pi^3 r^3$. SM particles are produced at $\Gamma_{RH}/H \equiv E_{RH}/E > 1$ where we have defined

$$E_{RH} \equiv c_{RH} h^2 m_{Pl} = 3.2 \left(\frac{h}{10^{-5}} \right)^2 10^4 GeV \quad (21)$$

and $c_{RH} \equiv \frac{\zeta(3)}{32\pi^4 r} \sqrt{\frac{90\Omega_{rel}}{g_r}} \simeq 10^{-4}$. If reheating takes place above $300 GeV$ we have $g_r^{SM} = 106.75$ and $\Omega_r = 1$ so that we can estimate $c_{RH} \simeq 10^{-4}$. Taking $h \leq 1$, so that $\alpha_h = h^2/4\pi < 0.1$, the maximum energy for reheating would be $10^{14} GeV$. However the limit for successful reheating scenario is much lower and it may be as low as $10 MeV$ [11, 12] corresponding to $h \simeq 10^{-8}$.

Therefore SM particles are produced at energies $E \leq E_{RH}$ and φ is in thermal equilibrium with SM particles with $T_\varphi = T_\gamma$ where T_γ is the photon temperature. As long as φ is relativistic $T_\varphi \propto T_\gamma$ and if it remains also in thermal equilibrium we have

$$\Omega_\varphi = \frac{g_\varphi}{g_{rel}^{SM}} \Omega_{rel}^{SM} \quad (22)$$

where g_{rel}^{SM} is the number of SM relativistic degrees of freedom and $g_\varphi = 1$.

B. Inflation-Dark Energy Unification

Let us now describe the inflation-dark energy unified picture with an explicit example. Again we start with two scalar fields coupled through eq.(17). To be specific we will choose $L_{int} = g \phi \varphi^3$ and a scalar potential $v(\phi)$ that inflates at high and low energies. As mentioned in the introduction in section IV the choice of the scalar potential is not important, there are a wide number of possibilities, and we choose to work with

$$v(\phi) = \frac{V_I}{2} \left(1 - \frac{2}{\pi} \arctan \frac{\phi}{f} \right) \quad (23)$$

which has two free parameters $E_I \equiv V_I^{1/4}$ and f with mass dimension. The potential in eq.(23) has two regions $\phi < -(2f/\pi)^{1/3}$ and $\phi > \sqrt{2}$ in which the slow roll conditions $|v'(\phi)/v(\phi)| \ll 1$ and $|v''(\phi)/v(\phi)| \ll 1$ are satisfied. The asymptotic expansions of $v(\phi)$ in these two regions are (see appendix B)

$$v(\phi) = \begin{cases} V_I \left(1 + \frac{f}{\pi\phi} \right) & \text{for } \phi < -f \\ \frac{V_I f}{\pi\phi} & \text{for } \phi > f \end{cases} \quad (24)$$

Inflation is associated with the high energy region $\phi < -(2f/\pi)^{1/3}$ with $v(\phi) \simeq V_I = E_I^4$ and dark energy with the region $\phi > \sqrt{2}$ with $v(\phi) \simeq \frac{V_I f}{\pi\phi}$. We determine the two free parameters in eq.(23) with the constraints coming from inflation $\frac{\delta\rho}{\rho} = 5.3 \times 10^{-4}$ and from dark energy

density $\rho_{DE} = 3H_o^2\Omega_{DE}$. Taking $\phi_o \simeq \sqrt{2}$ gives $E_I \simeq 100 \text{ TeV}$ and $f = \phi_o v_{DE}/V_I \simeq 10^{-39} \text{ eV}$ where we have reintroduced the correct mass units in E_I and f .

After inflation we want to reheat the universe with the SM particles. To achieve this we couple ϕ and φ via the interaction term $L_{int} = g\phi\varphi^3$ and φ with SM particles as in eq.(20). The ϕ field decays into φ via the process $\phi \rightarrow \varphi + \varphi + \varphi$ with a decay rate $\Gamma_d = \frac{g^2 m_\phi}{(2\pi)^3 72}$ given in eq.(18). This process starts immediately after inflation with $H \simeq E_I^2$. The maximum value of Γ_d/H is when m_ϕ is also at its maximum at $\phi \sim f$ giving $\Gamma_d/H \simeq 10^{36}$. Notice that $\Gamma_d \simeq 10^9$ at its maximum and the lifetime $\tau_\phi = 1/\Gamma_d$ of the ϕ particles is such that $\tau/\tau_{pl} \ll 1$ where τ_{pl} is the Planck time. Therefore all ϕ particles decay and at the end of the reheating and one has $\Omega_\phi = 0$ and $\Omega_\varphi + \Omega_{SM} = 1$. SM particles are produced through the interaction with the φ field via the interaction given in eq.(20) and described in section IV A. This process takes place for energies $E \leq E_{RH} \equiv c_{RH} h^2$ given in eq.(21) with $\Gamma_{RH}/H \geq 1$ and therefore φ and SM particles are in thermal equilibrium giving $\Omega_\varphi = \frac{g_\varphi}{g_{rel}} \Omega_{rel}^{SM}$ as long as φ remains relativistic. As discussed in section III as long as $E_{RH} > E > E_{gen} \equiv c_{gen} g^2$ (c.f eq.(15)) the universe contains the SM particles plus φ and at energies $E \leq E_{gen}$ the ϕ particles starts to be produced via the process $\varphi + \varphi \leftrightarrow \varphi + \phi$ with a decay rate $\Gamma_{gen} = \langle \sigma_{gen} v \rangle n_\varphi$. The inflation-dark energy unification scheme involves three different quantum processes: The ϕ decay into φ , the SM particles production and the late time ϕ generation. Only the first one (inflaton decay) depends on the choice of the potential $v(\phi)$, through its mass, the other two depend only on the size of the couplings g, h . These processes take place at energies

$$E_I, E_{RH} \equiv c_{RH} h^2, E_{gen} \equiv c_{gen} g^2 \quad (25)$$

where

$$\frac{\Gamma_d}{H} \geq 1, \quad \frac{\Gamma_{RH}}{H} \equiv \frac{E_{RH}}{E} \geq 1, \quad \frac{\Gamma_{gen}}{H} \equiv \frac{E_{gen}}{E} \geq 1 \quad (26)$$

and $c_{RH} \simeq 10^{-4}$ for a reheating temperature above 300 GeV and $c_{gen} \simeq 6 \cdot 10^{-4}$ if the ϕ is generated at energies below 1 MeV . We stress the fact that the ϕ generation at late times is due to the same interaction term $L_{int} = g\phi\varphi^3$ that gives the ϕ decay after inflation. The main difference is that at low energies E_{gen} the mass of ϕ is many orders of magnitude lower than its value at high energies E_I , i.e. $m_\phi(t_{RH}) \gg m_\phi(t_{gen})$ where t_{RH} and t_{gen} are the reheating and ϕ generation times respectively. We also require that $m_\phi(t_{gen}) \ll E_{gen}$ so that ϕ is relativistic at t_{gen} as in section III. Notice that in our model the value of the ϕ mass at generation time is $m_\phi(t_{gen}) \simeq 10^{-14} \text{ eV}$ (see appendix B) which is much smaller than $E_{gen} \simeq 1 \text{ eV}$. However at present times $m_\phi(t_o) \simeq 10^{-33} \text{ eV}$ which is a typical mass for a quintessence (dark energy) field.

After the production of the ϕ particles $v(\phi)$ is generated and the ϕ classical evolution will drive the expansion

of the universe as described in section III. Concluding we have shown with a simple example how the inflation-dark energy unification takes place. Of course it is possible to choose different scalar potentials $v(\phi)$ or interaction terms L_{int} that gives similar results.

C. E_{RH} and E_{gen} scales

The values of E_{RH} and E_{gen} do not depend on the choice of the potential $v(\phi)$ but only on the couplings g, h . The values of E_{RH} and E_{gen} are fixed in terms of the couplings g and h . In general g and h are free parameters that should give $E_I \geq E_{RH} > 10 \text{ MeV}$ and $E_{RH} \gg E_{gen} > E_o$ and are given by (see eq.(25))

$$E_{RH} \equiv c_{RH} h^2, \quad E_{gen} \equiv c_{gen} g^2 \quad (27)$$

and $c_{RH} \simeq c_{gen} \simeq 10^{-4}$. An interesting reduction of parameters is if we take $E_{RH} = \sqrt{E_{gen}}$ (in natural units) which gives $g = h^2$ and

$$E_{RH} = \left(\frac{E_{gen}}{1 \text{ eV}} \right)^{1/2} 5 \times 10^4 \text{ GeV} \quad (28)$$

Notice that this choice of g, h implies a low reheating temperature and for E_{gen} as small as $E_o \sim 10^{-3} \text{ eV}$ we have $E_{RH} \geq 1.5 \text{ TeV}$. If we set

$$g = h^2 = q \frac{E_I}{m_{pl}} = 4 \left(\frac{q}{100} \right) \left(\frac{E_I}{100 \text{ TeV}} \right) 10^{-12}, \quad (29)$$

with q a proportionality constant, we have

$$E_{gen} = \left(\frac{q}{100} \right)^2 \left(\frac{E_I}{100 \text{ TeV}} \right)^2 26 \text{ eV} \quad (30)$$

$$E_{RH} = \left(\frac{q}{100} \right) \left(\frac{E_I}{100 \text{ TeV}} \right) \text{ TeV}. \quad (31)$$

The fine structure constants associated to the two couplings are $\alpha_g \equiv \frac{g^2}{4\pi}, \alpha_h \equiv \frac{h^2}{4\pi}$ and cross sections σ_g, σ_{RH} are then

$$\alpha_g = \left(\frac{q}{100} \right)^2 \left(\frac{E_I}{100 \text{ TeV}} \right)^2 10^{-24} \quad (32)$$

$$\alpha_h = 3 \left(\frac{q}{100} \right) \left(\frac{E_I}{100 \text{ TeV}} \right) 10^{-13} \quad (33)$$

The cross section for the generation process are $\sigma_{gen} = g^2/(32\pi E_{gen}^2)$ and $\sigma_{RH} = h^2/(32\pi E_{RH}^2)$ giving

$$\sigma_{gen} = \frac{1}{32\pi c_{gen}^2 g^2} = \left(\frac{100}{q} \right)^2 \left(\frac{100 \text{ TeV}}{E_I} \right)^2 0.1 \text{ pb} \quad (34)$$

$$\sigma_{RH} = \frac{1}{32\pi c_{RH}^2 h^2} = \left(\frac{100}{q} \right) \left(\frac{100 \text{ TeV}}{E_I} \right) 10^{-11} \text{ pb} \quad (35)$$

We find the relationship between g and h in eq.(29) interesting but of course it does not need to hold since in principle g, h and therefore E_{gen}, E_{RH} are independent from each other and eqs.(30) and (31) are equivalent to eqs.(15) and (21), respectively.

V. PHENOMENOLOGY

In this section we summarize the main phenomenological consequences of the dark energy quantum generation. Let us first discuss the consequences of having the relativistic field φ . If φ is not contained in the SM then it represent an extra relativistic degree of freedom. CMB temperature anisotropies as well as SDSS and 2dF Large Scale galaxy clustering, Lyman- α absorption clouds, type Ia Supernovae luminosity distances and BAO data, can be used to determinate the value of the effective relativistic degrees of freedom, usually described in terms of the effective number of neutrinos N_ν^{eff} . The value of N_ν^{eff} affects the matter-radiation equality epoch and thus the ISW effect, so CMB anisotropies are sensitive to deviations from the standard cosmological model value of $N_\nu^{eff} \simeq 3.04$. Analysis of the WMAP data combined with other cosmological data sets allows for values of N_ν^{eff} different from its standard model value. For example in [30] it is found $N_\nu^{eff} = 4.6^{+1.6}_{-1.5}$ at 95% c.l., consistently with other analysis [31]. Moreover BBN is also affected by N_ν^{eff} , because the number of relativistic degrees of freedom change the value of the expansion rate of the universe and than influence the expected primordial Helium abundance. The BBN bound is $N_\nu^{eff} = 3.1^{+1.4}_{-1.2}$ at 95% c.l. [19, 30, 33], that seems to be more stringent than bounds coming from CMB data. Anyhow N_ν^{eff} can evolve from the BBN epoch at $T \sim 1\text{MeV}$ to the CMB decoupling era at $T \sim 1\text{eV}$ [33], so the different bounds coming from BBN and CMB are compatible. In our case the extra relativistic degree of freedom is represented by the scalar field φ that contributes to N_ν^{eff} an amount $\delta N_\nu^{eff} = \frac{4}{7} \left(\frac{T_\varphi}{T_\nu} \right)^4$. If the φ decouples from radiation before neutrinos, one has $T_\varphi \leq T_\nu$ and $\delta N_\nu^{eff} \leq 4/7 \simeq 0.57$, that is in full agreement with both BBN and CMB data. If φ is coupled with photons one has $\delta N_\nu^{eff} = \frac{4}{7} \left(\frac{T_\gamma}{T_\nu} \right)^4 \simeq 2.2$ that shows some tension with BBN data but is compatible with CMB data. In any case the existence of the extra relativistic degree of freedom coming from the φ field is consistent and apparently favored by cosmological data.

Another important phenomenological aspect of the dark energy generation model, concerns the coupling of the φ field with SM particles. In principle φ may be coupled with electrons, baryons and photons, but strong limits on the strength of such couplings comes from astrophysical considerations and accelerator physics. In fact if coupled with electrons, the φ particles are produced in stars and this fact affects the evolution of the stars, as studied in [37]. The coupling strength between φ and electrons should satisfy the condition $\alpha_{\varphi ee} < 0.5 \times 10^{-26}$. In the model that we present, SM particles are produced at $E_{RH} = c_{RH} h^2 m_{Pl} \geq 10\text{MeV}$ that gives $h \geq 10^{-8}$ and a fine structure constant $\alpha = h^2/4\pi \geq 10^{-17}$, therefore the φ field cannot be coupled with electrons. If coupled with baryons, the massless scalar field φ

could also generate long range forces [38] with possible observable consequences at astrophysical and cosmological level. The upper bound coming from long range force experiments is $\alpha_{\varphi B} < 10^{-47}$ [39] thus the φ coupling with baryons should be excluded. If the φ field is coupled with photons via the axion-like interaction term $L_{int} = \frac{g_{\varphi\gamma\gamma}}{4} \varphi F_{\mu\nu} \tilde{F}^{\mu\nu} = -g_{\varphi\gamma\gamma} \varphi \mathbf{E} \cdot \mathbf{B}$ with $g_{\varphi\gamma\gamma} < 10^{-10} \text{GeV}^{-1}$ [37]. The bound on the transition rate is $\Gamma_{\varphi\gamma\gamma} = \frac{g_{\varphi\gamma\gamma}^2 m_\varphi^3}{64\pi} \lesssim 10^{-4} (m_\varphi/\text{eV})^3 \text{eV}$. Taking $H = c_H T^2/m_{pl}$ the field φ and photons are coupled for $T \lesssim T_{RH}^\gamma \simeq (m_\varphi/\text{eV})^{3/2} 10^2 \text{GeV}$ when $\Gamma_{\varphi\gamma\gamma}/H \geq 1$. Therefore if φ is coupled with photons, SM particles will be produced at low reheating temperatures of about $T_{RH}^\gamma \simeq 10^2 \text{GeV}$ for $m_\varphi \simeq 1\text{eV}$. The coupling with photons is not ruled out by experimental data.

In conclusion, the φ field could be coupled either to neutrinos, photons, neutral Higgs field or supersymmetric partners of the SM which are currently searched for at LHC. It is particularly interesting the case in which the φ is coupled with neutrinos. In the standard cosmological model the three neutrinos free-stream and interact only gravitationally. Free-streaming lowers the neutrino perturbations and introduce a source of anisotropic stress. On the contrary, if one or more neutrinos are coupled with the scalar field φ , the interacting neutrinos behave as a tightly-coupled fluid with density and velocity perturbations but no anisotropy [35]. This fact also affects the adiabatic sound speed c_s^2 , that is equal to $1/3$ for free-streaming neutrinos and it is $0 < c_s^2 \leq 1/3$ for tightly-coupled neutrinos. Therefore the coupling of the φ field with neutrinos produces many observable consequences on the Cosmic Neutrino Background (CNB) [36].

In alternative to the dark energy generation scheme presented in section III, one can generate the ϕ field without any auxiliary field φ but coupling ϕ directly with SM neutrinos via an interaction $L_{int} = \sqrt{g} \phi \bar{\nu} \nu$. Therefore one or more neutrinos will be tightly-coupled to the ϕ quintessence field and this will produced observable consequences on the CNB [36].

The ϕ sector of the quantum generation model has also an interesting phenomenology. For example the inflation dark energy unified model presented in section IV has low inflationary and reheating scale $E_I \simeq 100\text{TeV}$ and $E_{RH} \simeq 1\text{TeV}$ for $h \simeq 10^{-6}$. This low reheating energy does not affect the reheating efficiency and it also avoids gravitino overabundance problems. From a cosmological point of view, a low inflationary scale may also affect N_ν^{eff} as showed in [12], giving one more possible test of the unified model. Moreover interesting effects may be observed in accelerators, as for example at LHC, with energy scales not so far from the inflationary energy. Then, as discussed above, a rich phenomenology exists, that may be used to constrain or falsify cosmological models that make use of the dark energy generation mechanism.

VI. CONCLUSIONS

We will now present a summary and conclusions of our work. One of the main goals of this paper was to understand why the dark energy is manifested at such a late time. To achieve this we have presented a novel idea, the quantum generation of dark energy, giving a new interpretation of the late time emergence of DE in terms of a late time quantum production of the quintessence ϕ particles. We take a $2 \leftrightarrow 2$ quantum process between ϕ and a relativistic φ particles. The scale where the ϕ field is generated is dynamically determined by the condition $\Gamma/H = E_{gen}/E \geq 1$ giving an energy scale $E \leq E_{gen}$ with $E_{gen} = c_{gen} g^2 m_{pl}$ and $c_{gen} \simeq 6 \cdot 10^{-4}$. Therefore the smallness of E_{gen} is due to a small coupling g and for $g \simeq 10^{-12}$ gives a $E_{gen} \simeq 1 \text{ eV}$ and a cross section $\sigma_{gen} \simeq 1 \text{ pb}$. The acceleration of the universe is then due to the classical evolution of ϕ and determined by the scalar potential $v(\phi)$. We have described in section III a universe that initially contains no ϕ particles, i.e. $n_\phi = \Omega_\phi = \dot{\phi} = v(\phi) = 0$, and once the relativistic particles ϕ are produced they become a source term for the generation of the scalar potential $v(\phi)$. Once $v(\phi)$ has been produced the classical equation of motion gives the evolution of ϕ .

We show in section IV that it is possible to unify inflation and dark energy using the same quintessence field ϕ . To achieve the unification we required that the potential $v(\phi)$ has two flat regions, at high energy for inflation and low energy for dark energy. In this scenario, after inflation the field ϕ decays completely and reheats the universe with standard model particles. The universe expands then in a decelerating way dominated first by radiation and later by matter. At low energies the same interaction term that gives rise to the inflaton decay accounts for the re-generation of the ϕ field giving rise to dark energy. An important difference in the quantum process between ϕ and φ at high and low energies is the value of transition rate due to the size of the ϕ mass, $m_\phi^2(E_I) \gg m_\phi^2(E_o)$.

We presented in section IV a simple example on how the inflation-dark energy unification can be implemented. We used a potential $v = E_I^4(1 - \arctan[\phi/f])$ which is flat at high and low energies. The two parameters E_I, f are determined by the density perturbations $\delta\rho/\rho$ and the value of v_o at present time giving $E_I = 100 \text{ TeV}, f = 10^{-39} \text{ eV}$. The coupling g between ϕ and φ and the coupling h between φ and the SM particles are free parameters but can be taken as $g = h^2 = q E_I/m_{pl} = (q/100)(E_I/100 \text{ TeV})10^{-12}$ giving a reheating energy $E_{RH} = (q/100)(E_I/100 \text{ TeV}) \text{ TeV}$ and a generation energy $E_{gen} = (q/100)^2(E_I/100 \text{ TeV})^2 26 \text{ eV}$. The cross section between ϕ and φ is $\sigma_{gen} = g^2/32\pi E_{gen}^2 \simeq \text{pb}$ quite close to cross section of WIMP dark matter with nucleons $\sigma_w \simeq \text{pb}$ at decoupling. By fixing $g = h^2 = q E_I$ we have determined the coupling, which set the scales of reheating and ϕ re-generation, in terms of the inflation scale E_I and we can reduced the number of parameters.

Of course this is not the only possible choice of g and h . To conclude, we have presented a general framework to produce the fundamental quintessence field ϕ dynamically at low energies. The energy scale is fixed by the strength of the coupling and this offers a new interpretation of the cosmological coincidence problem: dark energy domination starts at such small energies because of the size of the coupling constant g . Finally, our approach allows for an easy implementation of inflation and dark energy unification with the standard long periods of radiation/matter domination.

APPENDIX A: EQUATIONS OF MOTION

Here we derive the system of differential eqs.(2)-(5) that rules the ϕ generation. Let us consider a FRW universe containing the ϕ field coupled with a second scalar field φ and let us write down the equations of motion of the ϕ and φ fields. In what follows we assume that both the ϕ and φ particles are relativistic. The $\phi(t, x)$ field can be divided into a classical background configuration $\phi_c(t)$ plus a perturbation $\delta\phi(t, x)$ corresponding to the quantum configuration of the ϕ field (ϕ particles), in such a way that we can write $\phi(t, x) = \phi_c(t) + \delta\phi(t, x)$. We choose $\phi(t, x)$ and $\delta\phi(t, x)$ as independent variables, stressing the fact that when $\delta\phi(t, x) \rightarrow 0$ one has $\phi(t, x) = \phi_c(t)$. The dynamic of the ϕ field is then deduced from its lagrangian density

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V_T(\phi, \varphi) \quad (\text{A1})$$

where $V_T = v(\phi) + B(\varphi) + v_{int}(\phi, \varphi)$, $v(\phi)$ and $B(\varphi)$ are the classical potentials of the ϕ and φ fields respectively and $v_{int}(\phi, \varphi) = -L_{int}(\phi, \varphi)$ where $L_{int}(\phi, \varphi)$ is the interaction lagrangian of the scalar fields ϕ and φ . Using $\nabla\phi(t, x) = \nabla\delta\phi(t, x)$ one has

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\delta\phi}{a^2} + \partial_\mu V_T(\phi, \varphi) = 0 \quad (\text{A2})$$

The perturbation $\delta\phi$ evolves as a scalar field with mass $m_\phi^2 = \partial_\phi^2 v(\phi)$. The corresponding quantum operator $\hat{\delta\phi}$ has the following expression

$$\hat{\delta\phi} = \int \frac{d^3k}{(2\pi)^3\sqrt{2E_k}} \left[a_k f(t) e^{-i\vec{k}\vec{x}} + a_k^\dagger f^*(t) e^{i\vec{k}\vec{x}} \right] \quad (\text{A3})$$

where k and E_k are respectively the wave number and the energy of the ϕ particles with $E_k^2 = |\vec{k}|^2/a(t)^2 + m_\phi^2$ [25],[29]. The physical momentum is $\vec{p} = \vec{k}/a(t)$ and the ϕ particles are relativistic so $E_k = |\vec{k}|/a(t) = |\vec{p}|$. For simplicity we assume that all the ϕ and φ particles have the same energy E_ϕ and E_φ respectively. For example this assumption is valid if the quantum particles are thermalized. In this case the phase space distributions are the Bose-Einstein distributions $f_\phi(E) = 1/(e^{E/T_\phi} - 1)$ and $f_\varphi(E) = 1/(e^{E/T_\varphi} - 1)$ and one can take the energies of

the ϕ and φ particles as the mean values $E_\phi = \bar{r} T_\phi$ and $E_\varphi = \bar{r} T_\varphi$ with $\bar{r} \simeq 2.7$. Moreover, in the case that the ϕ and φ fields are in thermal equilibrium one has $E_\phi = E_\varphi$. Therefore we estimate $\delta\phi^2$ as the average of $\delta\hat{\phi}^2$ on the quantum state $|N, E_\phi\rangle$ containing N_ϕ particles with energy E_ϕ , i.e. $\langle N, E_\phi | : \delta\hat{\phi}^2 : | N, E_\phi \rangle = \frac{n_\phi}{E_\phi}$, where the $::$ stands for normal ordering of creation and destruction operators. We have then $\delta\phi^2 = n_\phi/E_\phi$, where n_ϕ is the density number of ϕ particles. Thus we can write

$$-\frac{\nabla^2 \delta\phi}{a^2} = |\vec{p}|^2 \delta\phi = E_\phi^{3/2} \sqrt{n_\phi} \quad (\text{A4})$$

where we have used $E_k^2 = |\vec{p}|^2 = |\vec{k}|^2/a^2$ for relativistic particles. Substituting this expression in eq.(A2) we have

$$\ddot{\phi} + 3H\dot{\phi} + E_\phi^{3/2} \sqrt{n_\phi} + \partial_\phi v(\phi) + \partial_\phi v_{int}(\phi, \varphi) = 0. \quad (\text{A5})$$

Repeating the same considerations for the φ field one has

$$\ddot{\varphi} + 3H\dot{\varphi} + E_\varphi^{3/2} \sqrt{n_\varphi} + \partial_\varphi B(\varphi) + \partial_\varphi v_{int}(\phi, \varphi) = 0. \quad (\text{A6})$$

The dynamics of the quantum particles is governed by the Boltzmann equations. If $f_\phi(E, t)$ is the phase space distribution of the ϕ particles, one has $n_\phi = \int f_\phi(E, t) \frac{d^3 p}{(2\pi)^3}$, $\rho_\phi = \int E f_\phi(E, t) \frac{d^3 p}{(2\pi)^3}$ and $p_\phi = \int \frac{|\vec{p}|^2}{3E} f_\phi(E, t) \frac{d^3 p}{(2\pi)^3}$. In the same way one has $n_\varphi = \int f_\varphi(E, t) \frac{d^3 p}{(2\pi)^3}$, $\rho_\varphi = \int E f_\varphi(E, t) \frac{d^3 p}{(2\pi)^3}$ and $p_\varphi = \int \frac{|\vec{p}|^2}{3E} f_\varphi(E, t) \frac{d^3 p}{(2\pi)^3}$ where $f_\varphi(E, t)$ is the phase space distribution of the φ particles. The evolution of the phase space density $f_\phi(E, t)$ is governed by the Boltzmann equation $\hat{L}[f_\phi] = \hat{C}[f_\phi]$, where \hat{L} is the Liouville operator and in a FRM metric is $\hat{L}[f_\phi(E, t)] = E \partial_t f_\phi(E, t) - H |\vec{p}|^2 \partial_E f_\phi(E, t)$ and \hat{C} is the collision operator (see [28]). If the ϕ particles are relativistic one has

$$\dot{n}_\phi + 3Hn_\phi = \int \hat{C}[f_\phi(E, t)] \frac{d^3 p}{(2\pi)^3 E} + A \quad (\text{A7})$$

where for a process $a_1 + a_2 + \dots + a_n \leftrightarrow b_1 + b_2 + \dots + b_l$, with $a_n(b_l)$ initial (final) particles, one has

$$\begin{aligned} \int C[f_\phi(E, t)] \frac{d^3 p}{(2\pi)^3 E} &= - \int d\Pi_{a_1} \dots d\Pi_{a_n} d\Pi_{b_1} \dots d\Pi_{b_l} \\ &\times (2\pi)^4 |M_{ab}|^2 \delta^4(\Sigma_i^n P_{a_i} - \Sigma_j^l P_{b_j}) \\ &\times [f_{a_1}(E, t) \dots f_{a_n}(E, t) - f_{b_1}(E, t) \dots f_{b_l}(E, t)] \end{aligned} \quad (\text{A8})$$

with $d\Pi \equiv g d^3 p / (2\pi)^3 2E$, g are the internal degrees of freedom and $|M_{ab}|^2$ the transition scattering matrix of the process. Eq.(A8) is valid in absence of Bose condensation of Fermi degeneracy when $1 \pm f_i(E, t) \simeq 1$ [28]. In the same way one has

$$\dot{n}_\varphi + 3Hn_\varphi = \int \hat{C}[f_\varphi(E, t)] \frac{d^3 p}{(2\pi)^3 E} + Q \quad (\text{A9})$$

The terms A and Q introduced in eqs.(A7) and (A9) are necessary for the energy conservation as we will discuss below eqs.(A18) and (A19).

Let us consider the quadratic interactions $L_{int} = g \phi^2 \varphi^2$ or $L_{int} = g \phi \varphi^3$. In that case the ϕ field is generated via the $2 \leftrightarrow 2$ processes $\varphi + \varphi \leftrightarrow \phi + \phi$ or $\varphi + \varphi \leftrightarrow \varphi + \phi$ respectively. For simplicity we assume that the phase space distribution $f_\varphi(E, t)$ of the φ particles is peaked around the mean energy E_φ of the φ particles. Of course this is true in the case of thermalized particles. Therefore we take all the φ particles with the same energy E_φ and one has $\int f_\varphi(E, t) d\Pi_\varphi = \int f_\varphi(E, t) \frac{d^3 p_\varphi}{2E_\varphi (2\pi)^3} \simeq \frac{1}{2E_\varphi} \int f_\varphi(E, t) \frac{d^3 p_\varphi}{(2\pi)^3} = \frac{n_\varphi}{2E_\varphi}$. Moreover, from energy conservation it follows that the ϕ particles are produced with the same energy of the φ particles and we take $E_\phi = E_\varphi = E$, that is also valid when the ϕ and φ fields thermalize. Therefore the energy distribution of the ϕ particles is peaked around the mean energy E_ϕ and one also has $\int f_\phi(E, t) d\Pi_\phi = \frac{n_\phi}{2E_\phi}$. The transition rates for the considered processes are $\Gamma_{\varphi\varphi \rightarrow \phi\phi} = \Gamma_{\varphi\varphi \rightarrow \phi\varphi} = \Gamma_{\phi\varphi \rightarrow \varphi\varphi} = \langle \sigma_{gen} v \rangle n_\varphi \equiv \Gamma_{gen}$ and $\Gamma_{\phi\phi \rightarrow \varphi\varphi} = \langle \sigma_{gen} v \rangle n_\phi$, where $\sigma_{gen} = g^2/32\pi E^2$ is the cross section for a $2 \leftrightarrow 2$ relativistic particle process and v is the relative velocity [25]. Considering the process $\varphi\varphi \leftrightarrow \varphi\phi$ one has

$$\begin{aligned} \int C[f_\phi(E, t)] \frac{d^3 p}{(2\pi)^3 E} &= - \int C[f_\varphi(E, t)] \frac{d^3 p}{(2\pi)^3 E} = \\ &= \Gamma_{\varphi\varphi \rightarrow \phi\phi} n_\varphi^2 - \Gamma_{\phi\phi \rightarrow \varphi\varphi} n_\phi n_\phi = \langle \sigma_{gen} v \rangle n_\varphi (n_\varphi - n_\phi) \end{aligned} \quad (\text{A10})$$

and for the process $\varphi\varphi \leftrightarrow \phi\phi$ one has

$$\begin{aligned} \int C[f_\phi(E, t)] \frac{d^3 p}{(2\pi)^3 E} &= - \int C[f_\varphi(E, t)] \frac{d^3 p}{(2\pi)^3 E} = \\ &= \Gamma_{\varphi\varphi \rightarrow \phi\phi} n_\varphi^2 - \Gamma_{\phi\phi \rightarrow \varphi\varphi} n_\phi^2 = \langle \sigma_{gen} v \rangle (n_\varphi^2 - n_\phi^2) = \\ &= \langle \sigma_{gen} v \rangle (n_\varphi + n_\phi)(n_\varphi - n_\phi) \end{aligned} \quad (\text{A11})$$

We can write eqs.(A10) and (A11) in a compact form as

$$\begin{aligned} \int C[f_\phi(E, t)] \frac{d^3 p}{(2\pi)^3 E} &= - \int C[f_\varphi(E, t)] \frac{d^3 p}{(2\pi)^3 E} = \\ &= \tilde{\Gamma} (n_\varphi - n_\phi) \end{aligned} \quad (\text{A12})$$

where we have defined $\tilde{\Gamma} \equiv \langle \sigma_{gen} v \rangle n_\varphi$ for the process $\varphi\varphi \leftrightarrow \varphi\phi$ and $\tilde{\Gamma} \equiv \langle \sigma_{gen} v \rangle (n_\varphi + n_\phi)$ for the process $\varphi\varphi \leftrightarrow \phi\phi$. Note that $\tilde{\Gamma}$ is not necessarily a transition rate, but it accounts for the whole contribution of the two processes $\varphi\varphi \rightarrow \varphi\phi$ and $\varphi\phi \rightarrow \varphi\varphi$ in one case and $\varphi\varphi \rightarrow \phi\phi$ and $\phi\phi \rightarrow \varphi\varphi$ in the other case.

Therefore eqs.(A7) and (A9) now read

$$\dot{n}_\phi + 3Hn_\phi = \tilde{\Gamma} (n_\varphi - n_\phi) + A \quad (\text{A13})$$

$$\dot{n}_\varphi + 3Hn_\varphi = -\tilde{\Gamma} (n_\varphi - n_\phi) + Q. \quad (\text{A14})$$

The energy density and pressure of the system are

$$\rho_T = \frac{\dot{\phi}^2}{2} + \frac{\dot{\varphi}^2}{2} + V_T(\phi, \varphi) + \frac{E_\phi n_\phi}{2} + \frac{E_\varphi n_\varphi}{2} \quad (\text{A15})$$

$$p_T = \frac{\dot{\phi}^2}{2} + \frac{\dot{\varphi}^2}{2} - V_T(\phi, \varphi) + \frac{E_\phi n_\phi}{6} + \frac{E_\varphi n_\varphi}{6} \quad (\text{A16})$$

with $V_T(\phi, \varphi) = v(\phi) + B(\varphi) + v_{int}(\phi, \varphi)$. Note that the terms proportional to the number densities in eqs.(A15)

and (A16) comes from the terms $|\nabla\delta\phi|^2/a(t)^2 = E_\phi^2\delta\phi^2 = E_\phi n_\phi$ and $|\nabla\delta\varphi|^2/a(t)^2 = E_\varphi^2\delta\varphi^2 = E_\varphi n_\varphi$

Therefore eqs.(A15) and (A16), together with eqs.(A5),(A6),(A13) and (A14) give the energy conservation in the form

$$\begin{aligned} \dot{\rho}_T + 3H(\rho_T + p_T) &= -E_\phi^{3/2}\sqrt{n_\phi}\dot{\phi} + \frac{A E_\phi}{2} - \\ &- E_\varphi^{3/2}\sqrt{n_\varphi}\dot{\varphi} + \frac{Q E_\varphi}{2} = 0 \end{aligned} \quad (\text{A17})$$

Note that eq.(A17) should be valid in the case in which only one of the scalar fields ϕ and φ exists. For example, if the only ϕ field exists one has $v_{int}(\phi, \varphi) = 0$, $n_\varphi = 0$ and $Q = 0$ and from eq.(A17) one has

$$A = 2\sqrt{E_\phi n_\phi}\dot{\phi} \quad (\text{A18})$$

and in the case in which the only φ field exists one obtains

$$Q = 2\sqrt{E_\varphi n_\varphi}\dot{\varphi}. \quad (\text{A19})$$

Then eqs.(A5),(A6),(A13) and (A14) now read

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + E_\phi^{3/2}\sqrt{n_\phi} + \partial_\phi v(\phi) + \partial_\phi v_{int}(\phi, \varphi) &= 0 \\ \ddot{\varphi} + 3H\dot{\varphi} + E_\varphi^{3/2}\sqrt{n_\varphi} + \partial_\varphi B(\varphi) + \partial_\varphi v_{int}(\phi, \varphi) &= 0 \\ \dot{n}_\phi + 3Hn_\phi = \tilde{\Gamma}(n_\varphi - n_\phi) + 2\sqrt{E_\phi n_\phi}\dot{\phi} \\ \dot{n}_\varphi + 3Hn_\varphi = -\tilde{\Gamma}(n_\varphi - n_\phi) + 2\sqrt{E_\varphi n_\varphi}\dot{\varphi} \end{aligned} \quad (\text{A20})$$

The system in eqs.(A20) describe the dynamics of two coupled relativistic scalar fields with the same energy $E_\phi = E_\varphi = E$. Note that this system includes the quantum interaction between the quantum ϕ and φ particles through the term $\tilde{\Gamma}(n_\varphi - n_\phi)$ in the last two equations in the system (A20). Moreover the density numbers n_ϕ and n_φ generates a source term for the corresponding fields ϕ and φ in first two equations of the system (A20) and this source term is responsible of the generation of the classical potential $v(\phi)$ during the ϕ generation. It is useful to divide the energy density ρ_T in two terms $\rho_T = \rho_\phi + \rho_\varphi$ with $\rho_\phi = \rho_{1\phi} + \rho_{2\phi}$,

$$\rho_{1\phi} = \frac{\dot{\phi}^2}{2} + v(\phi) \quad \rho_{2\phi} = \frac{E_\phi n_\phi}{2} \quad (\text{A21})$$

and $\rho_\varphi = \rho_{1\varphi} + \rho_{2\varphi}$,

$$\rho_{1\varphi} = \frac{\dot{\varphi}^2}{2} + v_{int}(\phi, \varphi) + B(\varphi), \quad \rho_{2\varphi} = \frac{E_\varphi n_\varphi}{2}. \quad (\text{A22})$$

In the same way we can write the pressure of the system as $p_T = p_\phi + p_\varphi$ with $p_\phi = p_{1\phi} + p_{2\phi}$,

$$p_{1\phi} = \frac{\dot{\phi}^2}{2} - v(\phi), \quad p_{2\phi} = \frac{E_\phi n_\phi}{6} \quad (\text{A23})$$

and $p_\varphi = p_{1\varphi} + p_{2\varphi}$,

$$p_{1\varphi} = \frac{\dot{\varphi}^2}{2} - v_{int}(\phi, \varphi) - B(\varphi), \quad p_{2\varphi} = \frac{E_\varphi n_\varphi}{6}. \quad (\text{A24})$$

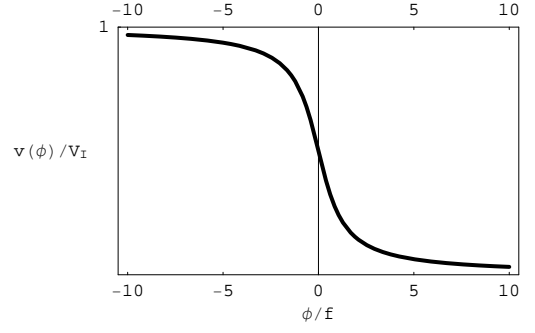


FIG. 9: Plot of the potential $v(\phi)$.

It is easy to check that these quantities verifies the following evolutionary equations

$$\begin{aligned} \dot{\rho}_{1\phi} + 3H(\rho_{1\phi} + p_{1\phi}) &= -\dot{\phi}\partial_\phi v_{int} - \dot{\phi}E_\phi^{3/2}\sqrt{n_\phi} \\ \dot{\rho}_{2\phi} + 3H(\rho_{2\phi} + p_{2\phi}) &= \frac{1}{2}E_\phi\tilde{\Gamma}(n_\varphi - n_\phi) + \dot{\phi}E_\phi^{3/2}\sqrt{n_\phi} \\ \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) &= \frac{1}{2}E_\phi\tilde{\Gamma}(n_\varphi - n_\phi) - \dot{\phi}\partial_\phi v_{int} \end{aligned} \quad (\text{A25})$$

and

$$\begin{aligned} \dot{\rho}_{1\varphi} + 3H(\rho_{1\varphi} + p_{1\varphi}) &= \dot{\phi}\partial_\phi v_{int} - \dot{\varphi}E_\varphi^{3/2}\sqrt{n_\varphi} \\ \dot{\rho}_{2\varphi} + 3H(\rho_{2\varphi} + p_{2\varphi}) &= -\frac{1}{2}E_\varphi\tilde{\Gamma}(n_\varphi - n_\phi) + \dot{\varphi}E_\varphi^{3/2}\sqrt{n_\varphi} \\ \dot{\rho}_\varphi + 3H(\rho_\varphi + p_\varphi) &= -\frac{1}{2}E_\varphi\tilde{\Gamma}(n_\varphi - n_\phi) + \dot{\varphi}\partial_\varphi v_{int} \end{aligned} \quad (\text{A26})$$

Notice that the sum of the first two equations of the systems in eqs.(A25) or (A26) gives the last equation, respectively, while the sum of the last equation in eqs.(A25) and (A26) gives the total energy evolution in eq.(A17) as it should due to energy-momentum conservation.

APPENDIX B: THE POTENTIAL $v(\phi)$

Let us study in more detail the properties of the potential

$$v(\phi) = \frac{V_I}{2} \left(1 - \frac{2}{\pi} \arctan \frac{\phi}{f} \right) \quad (\text{B1})$$

$$v'(\phi) = -\frac{V_I}{\pi f} \frac{1}{1 + (\phi/f)^2} \quad (\text{B2})$$

$$m_\phi^2 \equiv v''(\phi) = \frac{V_I}{\pi f^3} \frac{\phi}{(1 + (\phi/f)^2)^2} \quad (\text{B3})$$

where $E_I = V_I^{1/4}$ and f are parameters with mass dimensions. The ϕ mass is maximum at $|\phi| \simeq f$ with $m_\phi^2 \simeq V_I/f^2$ while v' is always negative. The asymptotic expansion of the potential in eq.(B1) for $|\phi/f| \gg 1$ is

$$v(\phi) \simeq \begin{cases} \frac{V_I}{2} \left(1 + \frac{f}{\pi\phi} \right) & \text{for } \phi < -f \\ \frac{V_I f}{\pi\phi} & \text{for } \phi > f \end{cases} \quad (\text{B4})$$

One can easily check that the slow roll conditions $|v'(\phi)/v(\phi)| \ll 1$ and $|v''(\phi)/v(\phi)| \ll 1$ are satisfied in

the two regions $\phi < -(2f/\pi)^{1/3}$ and $\phi > \sqrt{2}$. Therefore the region $\phi < -(2f/\pi)^{1/3}$ is associated with inflation at energies $E = \sqrt{\rho_\phi} \simeq E_I$. Dark energy is associated to the region $\phi > \sqrt{2}$ at energy $E = \sqrt{\rho_\phi} \simeq E_I(f/\phi_o)^{1/4} \simeq E_{DE} \simeq 3 \times 10^{-3} eV$ where we have chosen $\phi_o \simeq \sqrt{2}$ as the present time value of ϕ . We can fix the parameters E_I, f by imposing $\delta\rho/\rho = 5.3 \times 10^{-4}$ and from dark energy density $\rho_{DE} = 3H_o^2\Omega_{DE}$, this gives $E_I \simeq 100 TeV$ and $f \simeq 10^{-39} eV$, which gives a present time mass $m_\phi(t_o) \simeq 10^{-33} eV$ which is the standard value for quintessence field.

We can express the value of ϕ at the generation time, i.e. ϕ_{gen} , in terms of ϕ_o as $\phi_{gen} = \frac{v(\phi_{DE})}{v(\phi_{gen})} \phi_o = (\frac{E_{DE}}{E_{gen}})^4 \phi_o$ and if one choose $E_{gen} \simeq 1 eV$ one has $\phi_{gen} \simeq 10^{-12} \phi_o$. Therefore we can also compare the value of the mass m_ϕ at the reheating, generation and present times. At reheating one has $\phi \simeq f$ and $m_\phi(t_{RH}) \simeq \sqrt{V_I/f^2}$, at generation time one has $\phi_{gen} \gg f$ and $m_\phi(t_{gen}) \simeq \sqrt{\frac{V_I f}{\pi \phi_{gen}^3}}$ and at present time one has $\phi_o \gg f$ and $m_\phi(t_o) \simeq \sqrt{\frac{V_I f}{\pi \phi_o^3}}$. One has $m_\phi(t_{RH})/m_\phi(t_{gen}) \simeq \sqrt{\phi_{gen}/f} \gg 1$, therefore there are many orders of magnitudes of differ-

ence between the values of m_ϕ at the reheating and generation times, and this is why the reheating is obtained via the decay process $\phi \rightarrow \varphi + \varphi + \varphi$ of massive ϕ particles into relativistic φ particles, and the ϕ is generated at late times via a $2 \leftrightarrow 2$ process between relativistic particles.

We can also estimate the number of e-folds during inflation as $N = \ln \frac{a_f}{a_i} = -\int_{\phi_i}^{\phi_f} \frac{v(\phi)}{v'(\phi)} d\phi$. During inflation one has $\phi \leq -(2f/\pi)^{1/3} < f$, therefore we can use the second asymptotic expansion in eq.(B4) to write

$$N = -\int_{\phi_i}^{\phi_f} \frac{\pi \phi^2}{f} = \frac{\pi}{3f} (\phi_f^3 - \phi_i^3) \quad (B5)$$

Therefore if one require a minimum number N_m of e-folds during inflation, inflation must start at $\phi_i \leq (\phi_f^3 - f N_m / \pi)^{1/3} \simeq -(f/\pi)^{1/3} (N_m + 2)^{1/3}$ for $\phi_f \simeq -(2f/\pi)^{1/3}$. Note that a reasonable number of e-folds $N_m \simeq 50 - 100$ is easily achieved in the interval $\phi \in [-(f/\pi)^{1/3} (N_m + 2)^{1/3}, -(2f/\pi)^{1/3}]$ of width $\Delta\phi \sim \phi \sim f^{1/3}$.

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